Spatial Equilibrium:
Roback 1982 and Glaeser and Gottlieb 2009

Nathan Schiff
Shanghai University of Finance and Economics

Graduate Urban Economics, Week 4
March 13, 2017
Administration

- I have a homework exercise on monocentric city with Cobb-Douglas preferences. It’s good practice if you have time.
- Guanfu will present some of his work today between periods
- Do any grad students want to present work on spatial equilibrium in China?
How do we price negative effect of pollution?

Most cost-benefit analyses of “bads”, like pollution, need some measure for negative effect of pollution.

In order to evaluate policy we usually calculate a marginal willingness to pay; this can be compared to cost of implementing some pollution reduction policy.

So, if you wanted to quantify the negative effect of pollution on Beijing, how would you do it?

What would you try to measure? How would you report your results?

What method would you use to identify this effect? What data would you use?
Spatial Equilibrium Framework

A very useful method for estimating the value (or cost) of location specific factors is to try and infer this value from the decision of mobile agents.

In the spatial equilibrium framework workers have utility over tradable goods, non-tradable goods (ex: housing), and the location specific factors (“amenities”).

Since utility must be equal across locations, wages and housing prices adjust to make workers indifferent.

Therefore we can infer the value of amenities from wages and housing prices.
Wages, Rents, and the Quality of Life, JPE 1982

Very famous paper from Roback’s 1980 thesis, 2700 citations

Not only important to Urban Economics literature but also quite important to Labor (local labor markets—see references on last slide), Environmental Economics (pricing of environmental amenities), Trade (markets and industry concentration), and Development (migration)

Many theoretical extensions to consider heterogeneous agents, tax policy, agglomeration and congestion

Recent work uses framework in quite rigorous empirical estimation of quality of life and city wage differentials (see Albouy, “Are Big Cities Bad Places to Live? Estimating Quality of Life across Metropolitan Areas.”)
Workers

Identical workers with cost-less migration, each supplies one unit of labor

Different cities have different *exogenous* amenities (ex: warm climate, natural beauty, clean air), denoted $s$

Worker utility is function of $s$, consumption of composite commodity $X$ (numeraire, paid with wage $w$), and consumption of land $l^C$ (rented at $r$)

$$\max_{x,l^c} U(x, l^C; s) \text{ s.t. } x + r \cdot l^C = w + l$$  \hspace{1cm} (1)

Free migration ensures spatial equilibrium condition of equal utility:

$$V(w, r; s) = k$$  \hspace{1cm} (2)

Assume amenity increases utility: $V_s = \partial V / \partial s > 0$
Firms

Firms produce $X$ with CRS production function $X = f(L^p, N; s)$, where $N$ is number of workers and $L^p$ is land used in production.

In equilibrium the unit cost (CRS) must equal price of $X$ (assumed to be 1); if not firms could relocate to more profitable cities, which would increase factor prices (land, labor) in those cities until equilibrium is reached.

$$C(w, r; s) = 1$$  \hspace{1cm} (3)

Let $C_w = N/X$ and $C_r = L^p/X$; the amenity may be unproductive $C_s < 0$ or productive $C_s > 0$.

An amenity that is positive for consumers (clean air) may be unproductive for firms (clean air regulation may require expensive non-polluting technology).
Equilibrium

In equilibrium wages and rents adjust so that workers are indifferent across locations:

\[ V(w, r; s) = k \]  \hspace{1cm} (2)

\[ C(w, r; s) = 1 \]  \hspace{1cm} (3)

\[ L^p + N \times l^c = L \] \hspace{1cm} (Land Constraint)

Can use equations 2) and 3) to determine wages \( w \) and rent \( r \)

Easiest way is to graph isocost curves \( (C = 1) \) and indifference curves \( (V = k) \) in wage \( w \) and rent \( r \) space

Can vary amount of amenity \( s \) to see effect on curves and solve for equilibrium levels of \( w \) and \( r \)
Benchmark Cases

In David Card’s lecture notes he considers some simple cases to build up the intuition before considering Roback’s Figure 1

- Say that the amenity has no effect on production \((C_s = 0)\) and one city has nicer weather than another, \(s_2 > s_1\), \(V_s > 0\). How will wages and rent compare across the two cities? Will both wages and rent adjust or is one enough?

- What if the amenity has no effect on utility but does lower the cost of production: \(s_2 > s_1\), \(C_s < 0\), \(V_s = 0\)? For example, one city has access to cheap hydroelectricity (from an ugly river).

- Consider again a consumer amenity but for the case where not only does the amenity have no effect on production, but also land is not even used in production.
Consumer Amenity Only

\[ V_1 = V(w, r; s_2) = k \]
\[ V_2 = V(w, r; s_1) = k \]
\[ C = C(w, r) = 1 \]
\[ C_s = 0 \]

\[ s_2 > s_1, V(w_0, r_0; s_2) > V(w_0, r_0; s_1) \]
Producer Amenity Only

\[ V = V(w,r) = k \]
\[ V_s = 0 \]
\[ C_1 = C(w,r;s_1) = 1 \]
\[ C_2 = C(w,r;s_2) = 1 \]
\[ s_2 > s_1, \ C(w_0,r_0;s_1) > C(w_0,r_0;s_2) \]
Consumer Amenity, No Land in Production

\[ V_1 = V(w, r; s_2) = k \]
\[ V_2 = V(w, r; s_1) = k \]
\[ C = C(w) = 1 \]
\[ C_s = 0 \]
\[ C_r = 0 \]
\[ s_2 > s_1, \ V(w_0, r_0; s_2) > V(w_0, r_0; s_1) \]
Consumer Amenity, Producer Disamenity

What if $s$ is valued by consumer but increases cost of producer?

$$V(w, r; s_2)$$

$$C(w, r; s_1)$$

$$C(w, r; s_2)$$
Equilibrium Wage and Rent

If $s$ is valued by consumers but a disamenity for producers then wages are lower in $s_2$ city but rent may be higher or lower.

Similarly, if $s$ is valued by producers but a disamenity for consumers (ex: low pollution standards) then rent will be higher in low $s$ city but wages are uncertain.

Differentiate both equilibrium conditions, $V(w, r; s) = k$ and $C(w, r; s) = 1$ w.r.t. $s$:

$$\frac{dw}{ds} = \frac{-V_s \cdot C_r + C_s \cdot V_r}{V_w \cdot C_r - V_r \cdot C_w}$$

$$\frac{dr}{ds} = \frac{-V_w \cdot C_s + V_s \cdot C_w}{V_w \cdot C_r - V_r \cdot C_w}$$

Denominator is always positive (next slide) so if $V_s > 0$ and $C_s > 0$ then $dw/ds < 0$ but $dr/ds \nless 0$.
Using Roy’s Identity for Consumer Land Demand

\[ V(r, w; s) = \max_{x, l^c} U(x, l^c; s) \text{ s.t. } x + r \cdot l^c = w + l \]  \hspace{1cm} (1)

Let \( \lambda \) be marginal utility of additional unit of income, then:

\[ \frac{\partial V}{\partial w} = V_w = \lambda \]
\[ \frac{\partial V}{\partial r} = V_r = -\lambda \cdot l^c \]

This gives us \( V_r = -V_w \cdot l^c \), which we also know from Roy’s identity (ratio of derivative of indirect utility w.r.t. price and w.r.t. wealth is equal to Marshallian demand)

Then since \( C_r = L^p \) and \( C_w = N/X \), we know that

\[ V_w \cdot C_r - V_r \cdot C_w = V_w \cdot L^p - (-V_w \cdot l^c) \cdot (N/X) = V_w \cdot L/X > 0 \]
Quality of Life Derivation

Using the equilibrium condition we can infer the value of the amenity from changes in wages and rents

\[
\Omega(s) = V(w(s), r(s), s) = k
\]

\[
\frac{d\Omega(s)}{ds} = \frac{\partial V}{\partial w} \frac{dw}{ds} + \frac{\partial V}{\partial r} \frac{dr}{ds} + \frac{\partial V}{\partial s} = 0
\]

Using Roy’s identity we get:

\[
V_s = -V_w \frac{dw}{ds} + V_w l^c \frac{dr}{ds}
\]

\[
p_s^* \equiv \frac{V_s}{V_w} = l^c \frac{dr}{ds} - \frac{dw}{ds}
\]

This \(p_s^*\) is the marginal willingness to pay for an additional unit of the amenity \(s\)
Quality of Life and Valuing Amenities

\[ p_s^* \equiv \frac{V_s}{V_w} = l^c * \frac{dr}{ds} - \frac{dw}{ds} \]  

(5)

Albouy (2012) takes the total differential of the spatial equilibrium equation and log-linearizes around national averages of wages and prices; this gives him an index of quality of life that does not require choosing amenities (basically a residual from wages and prices for traded and non-traded goods)

The application of Roback (and Glaeser Gottlieb) is to value specific amenities (denote with \( z \)) by estimating \( \frac{dr}{dz} \) and \( \frac{dw}{dz} \) using regressions of rents on \( z \) and wages on \( z \) across cities with different amounts of \( z \)

Roback then multiplies the calculated weight \( p_z^* \) times the amount of the attribute \( z \) in a city, for all attributes \( z \in Z \), to get a measure of quality of life
Example Valuation

Say \( z \) is a bad amenity, such as crime or pollution; we want to know how people value a reduction in this bad

Roback re-writes eq5 as budget shares (easier estimation)

\[
p^*_z \equiv \frac{V_z}{V_w} = l^c \cdot \frac{dr}{dz} - \frac{dw}{dz} = w \left[ \frac{l^c \cdot r}{w} \cdot \frac{dr}{dz} \cdot \frac{1}{r} - \frac{dw}{dz} \cdot \frac{1}{w} \right] \\
p^*_z \cdot \frac{w}{w} = k_l \left( d \log r - d \log w \right) = k_l \cdot \gamma_r - \gamma_w
\]

In the above eq. \( k_l \) is the share of budget spent on land

Then we take an estimate of \( k_l \), run regressions for \( \gamma \)'s, and plug back into eq 5:

\[
\log w_{ic} = x_i \beta + \gamma_w \cdot z_c + \epsilon_{ic} \\
\log r_c = \gamma_r \cdot z_c + \mu_c
\]
Roback’s Extensions

Roback then extends the basic model by introducing a non-tradable goods (housing) sector.

This sector also competes for land use; incorporating this sector allows the author to derive the effect of change in $s$ on utility as a function of house price changes and wages.

Glaeser and Gottlieb extend this set-up even further and look more deeply at empirical implications.
Glaeser Gottlieb 2009: Introduction

Paper starts out by noting that while research on economic growth tends to focus on cross-country differences, differences within countries are also quite large.

In US, productivity in most productive cities is more than three times productivity in lowest cities.

Further, population is very concentrated: 68% of Americans live in 1.8% of land (we’ve seen this stat before).

Authors use classic spatial equilibrium model (Rosen-Roback) to explain how to empirically study these issues.

Three key equilibrium conditions: 1) workers indifferent between locations 2) firms indifferent about hiring more workers 3) builders indifferent about building more housing supply.

After describing model they show several empirical applications.
Productivity and City Size

Figure 1. Productivity and City Size

Notes: Units of observation are Metropolitan Statistical Areas under the 2006 definitions. Population is from the Census, as described in the Data Appendix. Gross Metropolitan Product is from the Bureau of Economic Analysis.

The regression line is \( \log \text{GMP per capita} = 0.13 \ [0.01] \times \log \text{population} + 8.8 \ [0.1]. \)

\( R^2 = 0.25 \) and \( N = 363. \)
The slow migration response to local shocks does not imply that spatial equilibrium holds only over long periods. As long as house prices or rents can change quickly, the price adjustment suffices to maintain the spatial equilibrium. Glaeser, Scheinkman, and Shleifer (1995) use a spatial equilibrium model where migration responds slowly to shocks but the spatial equilibrium is always maintained because of housing price flexibility. This leads us to ask if this occurs in practice: Do housing costs actually move enough to equalize utility levels across space?

If anything, Glaeser and Gyourko (2006) find that there is too much housing price volatility relative to volatility in local incomes. More generally, measurement difficulties mean that it is quite difficult to reject the hypothesis that welfare levels are equalized across space. The difficulties of assessing expected housing price appreciation makes it difficult to measure expected housing costs.

Figure 2. Income Over Time

Notes: Units of observation are Metropolitan Statistical Areas under the 2006 definitions, using Metropolitan Divisions where applicable. Data are from the Census, as described in the Data Appendix. The regression line is \( \text{Income 2000} = 0.77 \times \text{Income 1970} + 3.75 \). \( R^2 = 0.60 \) and \( N = 363 \).
City Production of Tradable Good

City-level production function with two types of capital: 1) tradable $K_T$ with national exogenous price $\gamma_t$ and 2) non-tradable $K_N$ (ex: land) with local endogenous price $\gamma_N$

Assume that amount of non-traded capital is fixed at stock $\bar{K}_N$, then:

$$G_T = A_t^i \bar{K}_N^{\alpha \gamma} K_T^\alpha (1-\gamma) L^{1-\alpha}$$  \hspace{1cm} (GG1)

Taking FOC w.r.t. $L$ and $K_T$ (assume some price of output) gives (inverse) demand for labor:

$$\phi A_t^i \bar{K}_N^{\alpha \gamma} L^{-\alpha \gamma} = W^{1-\alpha (1-\gamma)}$$  \hspace{1cm} (GG2)

The term $\phi$ includes price of output, price of traded capital, and other constants
Worker Utility 1

Workers get utility from 1) traded goods $G_T$ 2) non-traded goods $G_N$ and 3) amenities $\theta_t$

Can write this as indirect utility function like $V(w, r; s)$, with $Y^i_t$ as income (wage), $P^i_t$ as price of non-traded good (housing), and amenities:

$$V(Y^i_t, P^i_t, \theta^i_t)$$

Authors note that spatial equilibrium condition implies constant utility: $V(Y^i_t, P^i_t, \theta^i_t) = U_t$

Then holding $\theta$ fixed and taking the total differential we have $dV = V_y dY + V_P dP = 0$, or

$$\frac{dY}{dP} = -\frac{V_P}{V_Y}$$

An increase in housing prices is associated with higher income, or higher incomes are offset by higher housing prices.
Authors assume Cobb-Douglas utility with amenity multiplier:

$$U = \theta^i_t G_T^\beta G_N^{1-\beta}$$

Then, optimizing and plugging demand equations back into utility function gives indirect utility ($\omega$ is constant):

$$V = \omega \theta^i_t W^i_t (P^i_t)^{\beta-1} \quad \text{(GG4)}$$

Setting this equal to country utility $U_t$ we can solve for wage and take logs:

$$\log(W^i_t) = \log(U_t) + (1 - \beta) \log(P^i_t) - \log(\theta^i_t) \quad \text{(GG5)}$$
Motivation Basic Roback Model Glaeser Gottlieb Model Applications

Housing Prices vs Income

\[ \frac{dY_t}{dt} = \left( \frac{VP}{VY} \right) \frac{dP_t}{dt} \]

where the ratio \( \frac{VP}{VY} \) equals the demand for the nontraded good. High income levels are offset by high prices.

Again, the Cobb–Douglas utility function is a natural way to empirically use the spatial equilibrium assumption. Under this assumption, utility can be written as:

\[ \theta_t = G_t \beta G_t N^{1-\beta} \]

which will equal \( \theta_t W_t (P_t)^{\beta-1} \) times a constant.

The spatial equilibrium assumption requires this to equal \( U_t \), the reservation utility within the country. This formulation suggests that

\[ \log(W_t) = \log(U_t) + (1-\beta) \log(P_t) - \log(\theta_t) \]

Figure 3 shows the relationship between the logarithm of median home prices and the logarithm of median household income across space. The coefficient is 0.34, which is quite close to the average share of expenditure on housing, or 1 - \( \beta \).

The final critical production sector concerns the making of nontraded goods, or homes. If we are interested in a truly static model, as in Roback (1982), it is natural to follow her assumption that nontraded goods are produced like traded goods with labor, traded capital, and nontraded capital. In this case, the production function might be

\[ H_t = F(K, L) \]

where \( H_t \) refers to productivity in this sector. We will assume that the traded capital here is the same as the traded capital.

Figure 3. Housing Prices and Income

Notes: Units of observation are Metropolitan Statistical Areas under the 2006 definitions. Data are from the Census, as described in the Data Appendix.

The regression line is \( \log\text{income} = 0.34 \ [0.02] \times \log\text{value} + 5.97 \ [0.22] \). \( R^2 = 0.46 \) and \( N = 363 \).
Production of Non-Tradable Sector

Again, assume there is labor $L$ and tradable and non-tradable capital in production of $H$

Tradable capital is same as in city production of tradable good, $K_T$

Non-tradable capital (land) is different: $Z_N$, fixed at $\bar{Z}_N$

$$G_N = H_t^i \bar{Z}_N^{\mu \eta} K_T^{\mu (1 - \eta)} L^{1 - \mu}$$  \hspace{1cm} (GG6)

Note: I think equation at top of p993 has typo (should be $K_T$, not $K_N$)

Term $H_t^i$ is non-tradable productivity multiplier
Output of Non-Tradable Sector (Housing)

Then with price $P_t^i$ profit maximization gives output as:

$$\left( (P_t^i)^{1-\mu \eta} H_t^i W^{(\mu-1)} \right)^{1/\mu \eta} \bar{Z}_N$$  \hspace{1cm} (GG7)

Can also solve for labor demand from this non-tradable sector, which authors note is $(1 - \mu)(1 - \beta) \times N_t^i$, where $N_t^i$ is city population.

This part is generally unclear but population can be determined by labor demand for two sectors.
Equilibrium

Authors do not clearly write equilibrium equations; however, given text we can write this as:

\[ L^i_{Tt}(W^i_t) + L^i_{Nt}(W^i_t, P^i_t) = N^i_t \] \hspace{1cm} (1)

\[ \left( (P^i_t)^{1-\mu \eta} H^i_t W^{(\mu-1)} \right)^{1/\mu \eta} \bar{Z}_N = (1 - \beta) W^i_t / P^i_t \] \hspace{1cm} (2)

\[ V(W^i_t, P^i_t, \theta^i_t) = \omega \theta^i_t W^i_t (P^i_t)^{\beta-1} = U_t, \forall i \] \hspace{1cm} (3)

\[ \sum_i N^i_t = N_t \] \hspace{1cm} (4)

Eq 1 sums workers in two sectors, Eq 2 equates output of non-tradable good with expenditure from local consumers, Eq 3 is spatial equilibrium, Eq 4 distributes labor over all markets (labor market clearing).

Note: slides from Joaquin Blaum (MIT presentation) were very useful in corroborating above equations and I mostly use his notation.
Equilibrium Solution

\[
\begin{align*}
\log(N_t^i) &= \kappa_N + \lambda_A^N \log(A_t^i K_N^{\alpha \gamma}) + \lambda_H^N \log(H_t^i Z_N^{\mu \eta}) + \lambda_\theta^N \log(\theta_t^i) \\
\log(W_t^i) &= \kappa_W + \lambda_A^W \log(A_t^i K_N^{\alpha \gamma}) + \lambda_H^W \log(H_t^i Z_N^{\mu \eta}) + \lambda_\theta^W \log(\theta_t^i) \\
\log(P_t^i) &= \kappa_P + \lambda_A^P \log(A_t^i K_N^{\alpha \gamma}) + \lambda_H^P \log(H_t^i Z_N^{\mu \eta}) + \lambda_\theta^P \log(\theta_t^i)
\end{align*}
\]

Following table shows how parameters from model feed into λ’s and can be used for comparative statics

- Increase in tradable good productivity increases population, wages, house prices
- Increase in housing construction productivity increases population, decreases wages and house prices
- Increase in amenities increases population but decreases wages and house prices
Parameter Table

<table>
<thead>
<tr>
<th>Equation parameters</th>
<th>Value of parameters in the baseline model</th>
<th>Value of parameters with agglomeration economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_N^A )</td>
<td>( \frac{\beta + \mu(1 - \beta)(1 - \eta)}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{\beta + \mu(1 - \beta)(1 - \eta)}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_N^H )</td>
<td>( \frac{(1 - \alpha + \alpha\gamma)(1 - \beta)}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{(1 - \alpha + \alpha\gamma)(1 - \beta)}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_B^A )</td>
<td>( \frac{1 - \alpha + \alpha\gamma}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{1 - \alpha + \alpha\gamma}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_B^H )</td>
<td>( \frac{-\alpha\gamma(1 - \beta)}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{-\alpha\gamma(1 - \beta)}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_B^W )</td>
<td>( \frac{-\alpha\gamma}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{-\alpha\gamma}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_P^A )</td>
<td>( \frac{\mu\eta}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{\mu\eta}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_P^H )</td>
<td>( \frac{-\alpha\gamma}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{-\alpha\gamma}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
<tr>
<td>( \lambda_P^W )</td>
<td>( \frac{(1 - \alpha)\mu\eta - (1 - \mu)\alpha\gamma}{(1 - \alpha)\eta + \alpha\gamma}\mu(1 - \beta + \alpha\beta\gamma) )</td>
<td>( \frac{(1 - \alpha + \omega)\eta\mu - (1 - \mu)(\alpha\gamma - \omega)}{(1 - \alpha + \omega)(1 - \beta)\eta\mu + (\mu + \beta - \mu\beta)(\alpha\gamma - \omega)} )</td>
</tr>
</tbody>
</table>
How to use this in estimation

Researchers often try to estimate effect of some variable ($X^i_t$) on productivity in traded sector, non-traded sector, or amenities

\[
\frac{\partial \log(A_t^i \bar{K}_N^\alpha \gamma)}{\partial X^i_t} = \delta_A \tag{2}
\]

\[
\frac{\partial \log(H_t^i \bar{Z}_N^\mu \eta)}{\partial X^i_t} = \delta_H \tag{3}
\]

\[
\frac{\partial \log(\theta_t^i)}{\partial X^i_t} = \delta_\theta \tag{4}
\]

This effect can be inferred by running regressions of population, wages, and prices on variable $X^i_t$
Inferring Effects

\[ \hat{b}_N = E[N^i_t|X^i_t] = \lambda_A^N \delta_A + \lambda_H^N \delta_H + \lambda_\theta^N \delta_\theta \]  \hspace{1cm} (5)

\[ \hat{b}_W = E[W^i_t|X^i_t] = \lambda_A^W \delta_A + \lambda_H^W \delta_H + \lambda_\theta^W \delta_\theta \]  \hspace{1cm} (6)

\[ \hat{b}_P = E[P^i_t|X^i_t] = \lambda_A^P \delta_A + \lambda_H^P \delta_H + \lambda_\theta^P \delta_\theta \]  \hspace{1cm} (7)

Or:

\[ \delta_\theta = (1 - \beta) \hat{b}_P - \hat{b}_W \]  \hspace{1cm} (8)

\[ \delta_A = \alpha \gamma \hat{b}_N + (1 - \alpha (1 - \gamma)) \hat{b}_W \]  \hspace{1cm} (9)

\[ \delta_H = \mu \eta \hat{b}_N + (1 - \mu \mu \eta) \hat{b}_W - \hat{b}_P \]  \hspace{1cm} (10)

Ex: to understand effect of crime on utility \( \partial \log(\theta^i_t)/\partial X^i_t = \delta_\theta \) we use estimates of house prices and wages on (exogenous) crime, plus need estimate of share of household expenditure on housing \( (1 - \beta) \) from literature

Important: notice that this cannot be inferred solely from change in house prices, must also look at change in wages
Rise of the “Sunbelt”

In US, fastest growing areas have warm climates

These areas, in the south and west of US, are known as the “sunbelt”

Why has population growth shifted to sunbelt?

1. Has productivity increased in South?
2. Have political institutions become more efficient (and less corrupt)?
3. Has advent of air conditioning made South more comfortable (amenities)?
4. Are people attracted to cheap housing, made possible by pro-building policies?
Population Growth and Climate

Figure 4. Population Growth and Temperature

Notes: Units of observation are Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Population data are from the Census, as described in the Data Appendix. Mean January temperature is from the City and County Data Book, 1994.

The regression line is Population growth = 0.0030 [0.0004] × Temperature + 0.02 [0.01].

\[ R^2 = 0.16 \text{ and } N = 316. \]
Explaining the sunbelt 1

Authors run regressions of population, wages, and house values on temperature with controls

Combine coefficients using model to look at effect of temperature on amenities, productivity, housing construction productivity

Find

- Effect of temperature on productivity: $-0.14$
- Effect of temperature on amenities: $+0.59$—people are willing to give up 0.59% of real wages for an additional degree F.
- Effect of temperature on wages: $-0.52$
Sunbelt Regressions

TABLE 3
SPATIAL EQUILIBRIUM

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean January temperature</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.19 (0.06)</td>
<td>0.60 (0.31)</td>
<td>−0.33 (0.10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean January temperature × year 2000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.001 (0.05)</td>
<td>−0.43 (0.11)</td>
<td>0.19 (0.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Year 2000 dummy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25 (0.02)</td>
<td>0.62 (0.06)</td>
<td>0.06 (0.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Individual controls</strong></td>
<td>Yes</td>
<td>—</td>
<td>Yes</td>
<td>Yes</td>
<td>—</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Housing controls</strong></td>
<td>—</td>
<td>Yes</td>
<td>—</td>
<td>—</td>
<td>Yes</td>
<td>—</td>
</tr>
<tr>
<td><strong>MSA fixed effects</strong></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>1,590,467</td>
<td>2,341,976</td>
<td>1,590,467</td>
<td>2,950,850</td>
<td>4,245,315</td>
<td>2,950,850</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.29</td>
<td>0.36</td>
<td>0.21</td>
<td>0.27</td>
<td>0.60</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Notes: Individual-level data are from the Census Public Use Microdata Sample, as described in the Data Appendix. Metropolitan-area population is from the Census, as also described in the Data Appendix. Mean January temperature is from the City and County Data Book, 1994, and is measured in hundreds of degrees Fahrenheit. Real wage is controlled for with median house value, also from the Census as described in the Data Appendix. Individual controls include age and education. Location characteristics follow Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Standard errors are clustered by metropolitan area.
Explaining the sunbelt 2

Repeat exercise but look at growth instead of levels

Find that in 1990’s temperature not correlated with increases in productivity or amenities but is correlated with increasing housing supply

Conclude that over longer period, rise of Sunbelt due to both increases in productivity and housing supply but not amenities

Emphasize that most expensive US cities had large housing price increases with very little population growth

Authors argue that more attention should be paid to housing supply as a driver of population growth
House Values and City Growth

Figure 5. House Values and City Growth

Notes: Units of observation are Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Data are from the Census, as described in the Data Appendix.
Agglomeration

Authors extend model by allowing productivity to be a function of population, similar to earlier papers in our class.

This can strengthen some effects but also reverse others, depending on parameters.

Example: increase in amenities increases population but that may now increase productivity, possibly increasing wages.
Final Note: Estimation of Agglomeration

Typically we estimate agglomeration economies by regressing log income on log population or log density.

Spatial equilibrium model shows that population is an outcome and thus must be endogenous: some omitted variable increasing productivity must increase population.

Most papers try to instrument for population with a historical variable which may be exogenous to current unobservable productivity shocks.

In context of GG model, this means either correlated with $H_t^i$ or $\theta_t^i$.

Problem is that neither gives true treatment effect of population on productivity because will depend on other parameters (share of production associated with nontraded capital, or share of production associated with labor plus nontraded capital).
References

1. David Card’s lecture notes on local labor markets: http://eml.berkeley.edu/webfac/card/e250a_f16/lecture8-2016.pdf

2. Albouy, David, “Are Big Cities Bad Places to Live? Estimating Quality of Life across Metropolitan Areas.”, Working Paper (under revision at JPE, see davidalbouy.net to download)
