Home Market Effect, Increasing Returns, and Economic Geography
Discussion of Krugman AER 1980, JPE 1991

Nathan Schiff
Shanghai University of Finance and Economics

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Trade and Urban Economics

Why study Trade papers in an Urban Economics class?

One focus of Trade is to explain what happens when trade barriers between two areas (countries, but also cities) decreases

This decrease in trade costs can have a similar effect to an increase in population size, an important focus in Urban Economics

Further, many Trade papers study the choice of industries and firms across regions; also a focus of Urban Economics

Tools of Trade models commonly used in Urban, including Krugman-style CES models but also more modern Melitz and Eaton-Kortum models

Important difference from International Trade: in Urban we typically study within a country: free migration, no currency effects, common capital costs (but not always–see Henderson papers on China)

Paul Krugman

Nobel Prize winner 2008

Famous for contributions to trade and urban economics; most important founder of “New Economic Geography” (NEG)

Nice essay on Paul Krugman’s contribution to urban economics: Behrens and Robert-Nicoud, Papers in Regional Science, 2009

Krugman AER 1980: 5541 cites (as of yesterday)

Glaeser (NYTimes): “[JPE 1991] is one of only two models that I insist that Harvard’s Ph.D. students in urban economics be able to regurgitate, equation by equation.”

Krugman and Urban Economics

One focus of Krugman's work is to explain the spatial concentration of population

However, unlike many urban models, he does not want to allow for some kind of production externality

No spillovers, matching, or "ideas in the air" in his models

Instead, he uses market size to explain concentration

Unlike Rosen-Roback or Monocentric City model, firms now need to consider productivity of a place *and* market size: what firms pay workers affects the market for their products

Most of his models use: 1) increasing returns to scale (fixed cost in production) 2) CES utility, which adds a scale effect to consumption

Scale Economies, Product Differentiation, Pattern of Trade

Remarkable paper that, along with Krugman 1979, showed trade can occur without comparative advantage and when similar products produced in both places

Key component is monopolistic competition framework: CES utility means consumers want all products from other area

CES also ensures tractability; most results are analytical

Gave theoretical explanation for "Home Market Effect":

- Larger markets have higher wages and prices
- Areas with dissimilar tastes specialize in industry with larger home market

AER 1980: Closed Economy

Consumers have CES utility over a set of varieties:

$$U = \sum_{i=1}^{n} c_i^{\theta} \text{ for } 0 < \theta < 1 \tag{1}$$

Each consumer provides one unit of labor, which is the only input into production

Goods are produced with an IRS production function over labor, $x_i = \frac{l_i - \alpha}{\beta}$, or:

$$l_i = \alpha + \beta * x_i \tag{2}$$

Total output x_i equals sum of consumption from all L individuals and all labor employed:

$$L = x_i / c_i \tag{3}$$

$$L = \sum_{i=1}^{n} \alpha + \beta * x_i \tag{4}$$

Consumer Decision Problem

Consumers maximize utility s.t. spending over all n goods is equal to wage w:

$$\mathcal{L} = \sum_{i=1}^{n} c_i^{\theta} - \lambda * (\sum_{i=1}^{n} p_i * c_i - w) \tag{4a}$$

This gives FOC:

$$\theta * c_i^{\theta-1} = \lambda * p_i \tag{5}$$

Krugman works directly with eq 5 but I think intuition is easier if we solve for demand without λ (m. util of income)

CES Demand 1

$$\theta * c_i^{\theta-1} = \lambda * p_i \tag{5}$$

Consider MRS between good *i* and some reference good 0:

$$\frac{c_i}{c_0} = \left(\frac{p_i}{p_0}\right)^{\frac{1}{\theta-1}} \tag{NS5a}$$

Then plugging into the budget constraint:

$$\sum_{i=1}^{n} p_i * c_i = \sum_{i=1}^{n} p_i * c_0 * \left(\frac{p_i}{p_0}\right)^{\frac{1}{\theta-1}} = w \tag{NS5b}$$

Solving for c_0 and plugging back into MRS yields:

$$c_i = c_0 * \left(\frac{p_i}{p_0}\right)^{\frac{1}{\theta-1}} = \frac{w * p_i^{\frac{1}{\theta-1}}}{\sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}}} \tag{1}$$

CES Demand 2

$$c_i = \frac{w * p_i^{\frac{1}{\theta-1}}}{\sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}}} \tag{2}$$

If we assume that there are many products then the effect of changing p_i has little effect on the denominator

In CES models we assume many products and thus elasticity of demand is constant; here it is $1/(1-\theta)$

We also can re-arrange the denominator as a price index P that represents a cost of living index, or the amount a consumer must spend to get indirect utility u

CES Price Index

Define a price index P as:

$$P = \left[\sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}} \tag{3}$$

Then demand can be written as:

$$c_i = \frac{w * p_i^{\frac{1}{\theta-1}}}{\sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}}} = \left(\frac{p_i}{P}\right)^{\frac{1}{\theta-1}} \frac{w}{P} \tag{4}$$

If we plug demand back into the original utility function we will find indirect utility is $V = w/P$; this means that P is the price from the expenditure function $e(P, u) = P * u$

Firm Profit Maximization

Firm profits are $\Pi_i = p_i * x_i - w * l_i$

Plugging in the cost function $l_i = \alpha + \beta * x_i$ and the output equality $x_i = c_i * L$ we have:

$$\Pi_i = L\left[p_i * c_i - w * \beta * c_i\right] - w * \alpha \qquad \text{(NS6a)}$$

Now plug in demand and maximize w.r.t. p_i (or c_i–firms are monopolists); again, we assume p_i does not affect P:

$$\max_{p_i} \Pi_i = L\left[p_i * \frac{w * p_i^{\frac{1}{\theta-1}}}{P} - w\beta * \frac{w * p_i^{\frac{1}{\theta-1}}}{P}\right] - w\alpha \qquad \text{(NS6b)}$$

From the FOC we see that firm's have a constant markup on wages:

$$p_i = \frac{w * \beta}{\theta} \qquad (7)$$

Equilibrium

Plug equilibrium price $p_i = \frac{w*\beta}{\theta}$ back into demand function to find:

$$c_i = \frac{w * \left(\frac{w\beta}{\theta}\right)^{\frac{1}{\theta-1}}}{P} = \frac{\theta}{\beta * n} \tag{NS8a}$$

With free entry profit is zero and thus

$$\Pi_i = L\left[p_i * c_i - w\beta * c_i\right] - w * \alpha = w\left(\frac{L(1-\theta)}{n} - \alpha\right) = 0 \tag{NS8b}$$

Interesting, *here* free entry profit does not depend on wages, why?

Equilibrium number of firms is thus:

$$n = \frac{L(1-\theta)}{\alpha} \tag{10}$$

Effect of Trade with NO Transportation Costs

Now imagine there are two identical regions except one has a larger population, $L <> L^*$, where $L*$ denotes the “foreign” region

What will happen to the equilibrium compared with autarky? Will they trade?

Interestingly, they will trade but very little changes. Equilibrium markup and output are the same as before. Number of firms is $n = L(1 - \theta)/\alpha$, so more firms in bigger region.

Individuals in home country spend $n/(n + n^*)$ on goods produced at home and wages are equal in both countries. Anything different?

Notice that compared to autarky consumers are better off through gains from variety

Effect of Trade with Transportation Costs

Iceberg trade costs: a constant percentage of the product $1 - g$ *melts* on way to other region so that only $g < 1$ percent arrives

How will this affect equilibrium?

Again, firm mark-up is same, $p = w * \beta/\theta$ and equilibrium number of firms follows same equation $n = L(1 - \theta)/\alpha$

But, wages will be higher in region with larger population!

Very mysterious result. Proved in paper using balance of payments: payments on imports must equal revenue from exports.

However, there is no closed form solution, makes intuition difficult.

Not trivial: consumers in larger region are better off both through greater variety at cheaper cost *and* because higher wage buys more imported goods

My attempt at intuition

We have three results to think about:

- In autarky, larger market has more firms but wages and prices can be equal in both markets (not separately identified)
- When regions can trade *without* transportation costs wages and prices can be the same in both markets. This we can think of as just merging the regions–there is no advantage to any region since goods are freely shipped
- With transportation costs, bigger region is more attractive to firms because they can sell to home market without transportation costs. Somehow this makes wages higher

Think about the profit equation: if p/w is always constant why do higher wages matter?

Higher wages affect fixed cost $w\alpha$. Now effect of additional firm on profits varies by region, even if count of firms same in both regions. Wages must be higher in more profitable region.

Home Market Effect with Two Industries

Next Krugman allows for two different types of industries where α consumers purchase α industry goods and β purchase β goods

This allows for different distributions of tastes, so home market effect now extends to types of industries

Using similar logic as before, Krugman shows that when tastes are dissimilar enough, countries will specialize in industries where they have a larger home market

If tastes are below this dissimilarity threshold, both regions produce both goods but region with larger home taste is still net exporter of good

Paul Krugman's most famous paper

Contributions of JPE 1991 according to Behrens and Robert-Nicoud:

1. understand how market size influences location choices and location choices influence market size (circular)
2. shows mechanism leading to persistence and path dependence
3. connects location theory to trade theory
4. provided a framework (NEG) applicable to many contexts (ex: taxes) and flexible to incorporate many extensions

Balwin et. al. also note that this paper has "catastrophic agglomeration," meaning a small change in an underlying parameter suddenly moves the system to a very different state. This is unlike most economics models, but commonly found in models of physical processes (ex: plate tectonics).

Main Questions of Paper

What is the main question of this paper?

Why is manufacturing concentrated and specifically, can we explain this with the interaction of transportation costs and economies of scale?

Sub-questions:

- How sensitive are concentration results to transportation costs?
- What is overall effect of divergence forces (access to bigger market, lower transportation) and convergence forces (competition among firms for fixed expenditure)?
- When will all manufacturing be concentrated in one region? Does the starting point matter?

Basic Setup

- There are two regions and two sectors, farming and manufacturing, which employ farmers and manufacturing workers
- Farmers are immobile but manufacturing workers can migrate costlessly across regions
- Trade: the farm good (numeraire) can be shipped to other region costlessly but manufacturing goods have iceberg transport costs: if you send 1 unit of a good from region 1 to region 2 it melts to only τ units when it arrives in region 2, $\tau < 1$
- Goal: characterize equilibrium in terms of manufacturing workers in each region; this will provide insight into determinants of concentration

Consumption

All residents (farmers and workers) have two-level utility function:

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

$$C_M = \left[\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

N is number of potential products, $\sigma > 1$; notice consumers have taste for variety

Why two-level? Parameter μ determines expenditure on manufacturing, key to concentration

Production

Farmers: one unit of farm labor produces one unit of agricultural good

Supply of farmers is fixed and equal in both regions: $(1-\mu)/2$

Workers are mobile but define L_1 and L_2 as workers in each region

$$L_1 + L_2 = \mu \tag{3}$$

Production function of manufacturing good i: $x_i = \frac{L_{Mi}-\alpha}{\beta}$, or

$$L_{Mi} = \alpha + \beta * x_i \tag{4}$$

Notice: fixed cost generates IRS and that will vary with wage rate

Optimal price

Symmetry will yield that price is the same for all varieties

Let p_1 be price of representative variety in region 1, w_1 manufacturing wage. Optimization gives:

$$p_1 = \frac{\sigma}{\sigma - 1}\beta * w_1 \tag{5}$$

Region 2 has similar equation, thus:

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} \tag{6}$$

Free entry gives zero profit: $\pi = 0$:

$$(p_1 - \beta * w_1)x_1 = \alpha * w_1 \tag{7}$$

Number of varieties

Zero profit condition and price ratio across regions $p_1/p_2 = w_1/w_2$ gives:

$$x_1 = x_2 = \frac{\alpha(\sigma - 1)}{\beta} \tag{8}$$

Notice that output of any good does *not* depend on any region specific variables

With output of each firm we can figure out labor requirement of each firm, which is $\alpha\sigma$

Then number of firms in a region is total labor divided by per-firm labor, $n_1 = L_1/\alpha\sigma$, and:

$$\frac{L_1}{L_2} = \frac{n_1}{n_2} \tag{6}$$

Elasticity of substitution measures economies of scale

Turns out that σ shows MPL/APL

MPL=$\frac{1}{\beta}$, APL=$\frac{\sigma-1}{\beta\sigma}$

MPL/APL=$\frac{\sigma}{\sigma-1}$

Smaller σ gives greater economies of scale (MPL/APL larger)

Why? As σ decreases consumer wants to consume more and more varieties ($\sigma = 1$ is Cobb-Douglas)

Low σ leads to more firms, lower output per firm, less labor per firm, lower APL

Note: this is a bit confusing since firms are more productive (lower AC) with *higher* σ; however, lower σ leads to greater agglomeration economies (same as models with CES production and intermediate input sharing)

Short-run Equilibrium

In short-run workers in each region can't migrate, want to look at wages

Define c_{11} as *total* consumption in region 1 of a representative region 1 good and c_{12} is region 1 consumption of representative region 2 good

Region 2 must ship $1/\tau$ units so that 1 unit arrives in region 1, thus region 1 consumers pay $p_2 * (1/\tau)$ for one unit

In region 1, ratio of demand for good 1 to good 2 is:

$$\frac{c_{11}}{c_{12}} = \left(\frac{p_1 \tau}{p_2}\right)^{-\sigma} = \left(\frac{w_1 \tau}{w_2}\right)^{-\sigma} \tag{10}$$

This equation comes from demand function and equation 6

Expenditure Ratios

Define z_{11} as ratio of total region 1 expenditure on region 1 goods to region 1 expenditure on region 2 goods

$$z_{11} = \left(\frac{n_1}{n_2}\right)\left(\frac{p_1\tau}{p_2}\right)\left(\frac{c_{11}}{c_{12}}\right) = \left(\frac{L_1}{L_2}\right)\left(\frac{w_1\tau}{2_2}\right)^{-(\sigma-1)} \tag{11}$$

1. A one percent increase in relative prices reduces quantities sold by σ (eq 10) but reduces value by only $\sigma - 1$; basically p is higher in $p * q$
2. As a region's *number* of goods increases overall expenditure share z_{11} also increases

Closing Model

To close model write equations for both regions where total income has to be equal to total expenditure

Define Y_1 and Y_2 as total income in a region

$$Y_1 = \frac{1-\mu}{2} + w_1 L_1 \tag{15}$$

Then we have:

$$w_1 * L_1 = \mu \left[\left(\frac{z_{11}}{1+z_{11}} \right) Y_1 + \left(\frac{z_{12}}{1+z_{12}} \right) Y_2 \right] \tag{13}$$

Short-run equilibrium results

Set of equations *determine* wages but no closed form solution (!); increases difficulty of model

Because many results echo 1980 AER paper he doesn't discuss in detail

Main focus in short-run is ratio of wages in regions w_1/w_2:

1. Increase in manufacturing employment raises utility (lowers variety-adjusted price index) because allows for greater number of firms
2. Increase in employment also can raise wages by more than proportionally increasing output: home market effect
3. However, also a competitive effect working in opposite direction–workers have to share limited amount of peasant expenditure

Long-run Equilibrium: Choice of Units

In long-run we allow workers to migrate

Krugman carefully chooses units to simplify this problem (but without mentioning this, ridiculously confusing!)

$\beta = \frac{\sigma-1}{\sigma}$ and $\alpha = \frac{\mu}{\sigma}$

Since $p_1 = \frac{\sigma}{\sigma-1}\beta * w_1$ and $n_1 = \frac{L_1}{\alpha*\sigma}$

This implies that $p_1 = w_1$ and $n_1 = L_1/\mu$

Long-run Equilibrium: Price Indices

With CES a price index is cost of purchasing one unit of *composite* good at optimal consumption of each variety

$$P = \left[\sum_{i=1}^{N} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{N1}$$

Price of consumption in region 1 includes local goods and imports

Let f be fraction of total workers in region 1, $f = L_1/\mu$, then using choice of units:

$$P_1 = \left[f w_1^{-(\sigma-1)} + (1-f) \frac{w_2}{\tau}^{-(\sigma-1)} \right]^{\frac{-1}{\sigma-1}} \tag{17}$$

Real Wages

Workers migrate based on real wages: what they can consume given region's nominal wage

To calculate real wages we need a cost-of-living index: cost of a given level of utility in a region

CES has a convenient form because expenditure measures utility: $P^{\mu} * P_A^{1-\mu}$

Then real wages are:

$$\omega_1 = \frac{w_1}{P_1^{\mu}} \tag{19}$$

Equilibrium Comparative Statics

Main goal is to explain concentration across regions: when will most workers concentrate in one region ("core") with small region ("periphery") versus more equal sized regions?

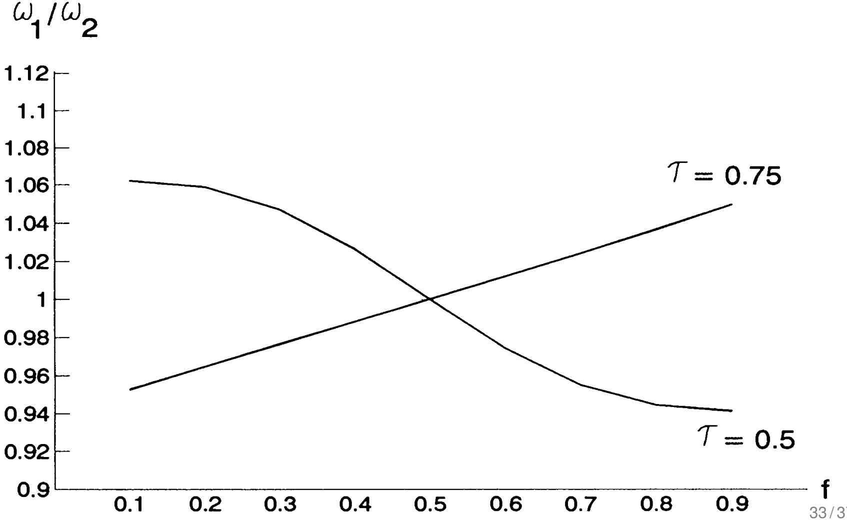
Key is to know how real wage ratio ω_1/ω_2 varies with share of labor force f; if real wage increases with migration then feedback effects will lead to core-periphery equilibrium

Set of non-linear equations makes analytical solution difficult; instead uses numerical exercises to illustrate main idea

Three parameters:

1. Share of consumer budget spent on composite good μ
2. Transportation cost τ–transportation cost increases when τ *decreases*
3. Elasticity of substitution σ, which can measure economies of scale: smaller σ greater economies of scale

Effect of concentration on wage ratio varies by transport cost



Interpretation

When will industry concentrate?

Depends on interaction of key parameters

Dispersion: high transportation cost, low manufacturing consumption share, weak economies of scale (large σ)

Concentration: low transportation cost, high manufacturing consumption share, large economies of scale (small σ)

Next figure draws boundaries for convergence (concentration): above the line leads to concentration

Boundaries for concentration equilibrium

Increasing τ is decreasing transportation cost; increasing μ is increasing concentration of manufacturing

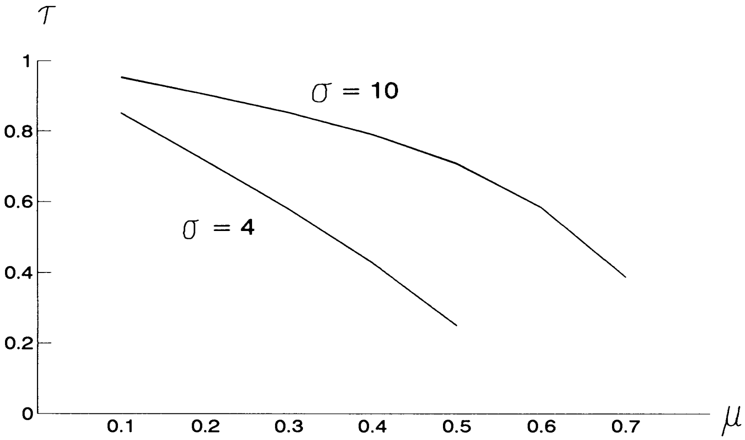


FIG. 3

When $\nu < 1$ manufacturing will concentrate

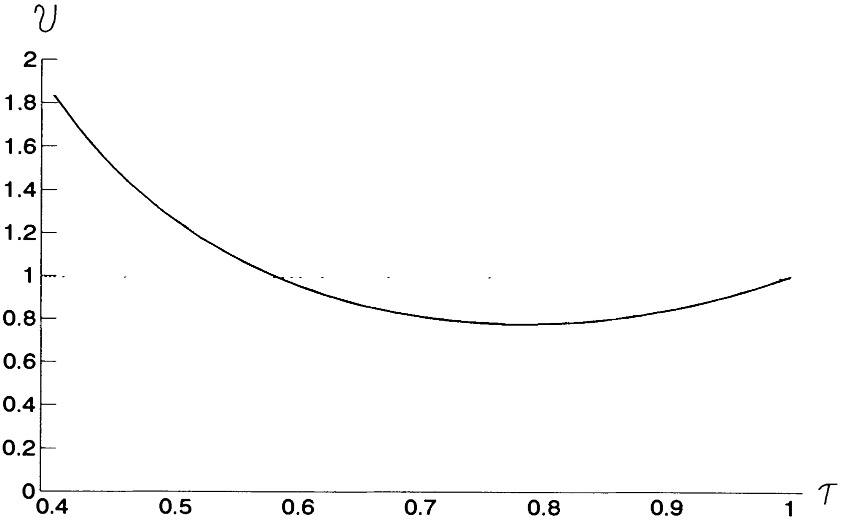


Fig. 2

Main Forces

Pecuniary Externalities: no direct spillovers in terms of productivity but larger markets have important effects (ex: raise demand, allow more varieties, increase utility)

IRS and taste for variety: leads to one firm per type

Home Market Effect: firms want to locate in larger markets

Can sell to domestic consumers without transportation costs (demand higher, price index lower)

This effect exists even without allowing for mobile workers; in Faber paper connecting two asymmetric regions leads to greater concentration in bigger region (firms ship goods to smaller region)

Reinforcement: given parameters, mobile workers can reinforce home market effect by moving to bigger region, increasing market size with own consumption