Home Market Effect, Increasing Returns, and Economic Geography

Discussion of Krugman AER 1980, JPE 1991

Nathan Schiff
Shanghai University of Finance and Economics

Graduate Urban Economics, Lecture 9
April 24, 2019
Trade and Urban Economics

Why study Trade papers in an Urban Economics class?

One focus of Trade is to explain what happens when trade barriers between two areas (countries, but also cities) decreases.

This decrease in trade costs can have a similar effect to an increase in population size, an important focus in Urban Economics.

Further, many Trade papers study the choice of industries and firms across regions; also a focus of Urban Economics.

Tools of Trade models commonly used in Urban, including Krugman-style CES models but also more modern Melitz and Eaton-Kortum models.

Important difference from International Trade: in Urban we typically study within a country: free migration, no currency effects, common capital costs (but not always—see Henderson papers on China).
Paul Krugman

Nobel Prize winner 2008

Famous for contributions to trade and urban economics; most important founder of “New Economic Geography” (NEG)

Nice essay on Paul Krugman’s contribution to urban economics: Behrens and Robert-Nicoud, Papers in Regional Science, 2009

Krugman AER 1980: 5541 cites (as of yesterday)

Glaeser (NYTimes): “[JPE 1991] is one of only two models that I insist that Harvard’s Ph.D. students in urban economics be able to regurgitate, equation by equation.”
Krugman and Urban Economics

One focus of Krugman’s work is to explain the spatial concentration of population

However, unlike many urban models, he does not want to allow for some kind of production externality

No spillovers, matching, or “ideas in the air” in his models

Instead, he uses market size to explain concentration

Unlike Rosen-Roback or Monocentric City model, firms now need to consider productivity of a place and market size: what firms pay workers affects the market for their products

Most of his models use: 1) increasing returns to scale (fixed cost in production) 2) CES utility, which adds a scale effect to consumption
Scale Economies, Product Differentiation, Pattern of Trade

Remarkable paper that, along with Krugman 1979, showed trade can occur without comparative advantage and when similar products produced in both places.

Key component is monopolistic competition framework: CES utility means consumers want all products from other area.

CES also ensures tractability; most results are analytical.

Gave theoretical explanation for “Home Market Effect”:
- Larger markets have higher wages and prices.
- Areas with dissimilar tastes specialize in industry with larger home market.
AER 1980: Closed Economy

Consumers have CES utility over a set of varieties:

$$U = \sum_{i=1}^{n} c_i^\theta \text{ for } 0 < \theta < 1$$ (1)

Each consumer provides one unit of labor, which is the only input into production.

Goods are produced with an IRS production function over labor, $x_i = \frac{l_i - \alpha}{\beta}$, or:

$$l_i = \alpha + \beta * x_i$$ (2)

Total output $x_i$ equals sum of consumption from all $L$ individuals and all labor employed:

$$L = x_i / c_i$$ (3)

$$L = \sum_{i=1}^{n} \alpha + \beta * x_i$$ (4)
Motivation
AER 1980
JPE 1991
Model Setup
Equilibrium

Consumer Decision Problem

Consumers maximize utility s.t. spending over all \( n \) goods is equal to wage \( w \):

\[
\mathcal{L} = \sum_{i=1}^{n} c_i^\theta - \lambda \left( \sum_{i=1}^{n} p_i \cdot c_i - w \right) \tag{4a}
\]

This gives FOC:

\[
\theta \cdot c_i^{\theta-1} = \lambda \cdot p_i \tag{5}
\]

Krugman works directly with eq 5 but I think intuition is easier if we solve for demand without \( \lambda \) (m. util of income)
CES Demand 1

\[ \theta \ast c_i^{\theta-1} = \lambda \ast p_i \]  \hspace{1cm} (5)

Consider MRS between good \( i \) and some reference good \( 0 \):

\[ \frac{c_i}{c_0} = \left( \frac{p_i}{p_0} \right)^{\frac{1}{\theta-1}} \]  \hspace{1cm} (NS5a)

Then plugging into the budget constraint:

\[ \sum_{i=1}^{n} p_i \ast c_i = \sum_{i=1}^{n} p_i \ast c_0 \ast \left( \frac{p_i}{p_0} \right)^{\frac{1}{\theta-1}} = w \]  \hspace{1cm} (NS5b)

Solving for \( c_0 \) and plugging back into MRS yields:

\[ c_i = c_0 \ast \left( \frac{p_i}{p_0} \right)^{\frac{1}{\theta-1}} = \frac{w \ast p_i^{\frac{1}{\theta-1}}}{\sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}}} \]  \hspace{1cm} (1)
If we assume that there are many products then the effect of changing $p_i$ has little effect on the denominator

In CES models we assume many products and thus elasticity of demand is constant; here it is $1/(1 - \theta)$

We also can re-arrange the denominator as a price index $P$ that represents a cost of living index, or the amount a consumer must spend to get indirect utility $u$
CES Price Index

Define a price index $P$ as:

$$ P = \left[ \sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}} \right] \frac{\theta-1}{\theta} $$

(3)

Then demand can be written as:

$$ c_i = \frac{w \times p_i^{\frac{1}{\theta-1}}}{n \sum_{i=1}^{n} p_i^{\frac{\theta}{\theta-1}}} = \left( \frac{p_i}{P} \right)^{\frac{1}{\theta-1}} \frac{w}{P} $$

(4)

If we plug demand back into the original utility function we will find indirect utility is $V = w/P$; this means that $P$ is the price from the expenditure function $e(P, u) = P \times u$
Firm Profit Maximization

Firm profits are $\Pi_i = p_i \times x_i - w \times l_i$

Plugging in the cost function $l_i = \alpha + \beta \times x_i$ and the output equality $x_i = c_i \times L$ we have:

$$\Pi_i = L \left[ p_i \times c_i - w \times \beta \times c_i \right] - w \times \alpha \quad \text{(NS6a)}$$

Now plug in demand and maximize w.r.t. $p_i$ (or $c_i$—firms are monopolists); again, we assume $p_i$ does not affect $P$:

$$\max_{p_i} \Pi_i = L \left[ p_i \times \frac{w \times p_i^{\theta - 1}}{P} - w \beta \times \frac{w \times p_i^{\theta - 1}}{P} \right] - w \alpha \quad \text{(NS6b)}$$

From the FOC we see that firm’s have a constant markup on wages:

$$p_i = \frac{w \times \beta}{\theta} \quad \text{(7)}$$
Plug equilibrium price \( p_i = \frac{w^* \beta}{\theta} \) back into demand function to find:

\[
c_i = \frac{w^* \left( \frac{w \beta}{\theta} \right)^{1 \theta - 1}}{P} = \frac{\theta}{\beta \ast n} \tag{NS8a}
\]

With free entry profit is zero and thus

\[
\Pi_i = L [p_i \ast c_i - w \beta \ast c_i] - w \ast \alpha = w \left( \frac{L(1 - \theta)}{n} - \alpha \right) = 0 \tag{NS8b}
\]

Interesting, here free entry profit does not depend on wages, why?

Equilibrium number of firms is thus:

\[
n = \frac{L(1 - \theta)}{\alpha} \tag{10}
\]
Effect of Trade with NO Transportation Costs

Now imagine there are two identical regions except one has a larger population, $L \leftrightarrow L^*$, where $L^*$ denotes the “foreign” region.

What will happen to the equilibrium compared with autarky? Will they trade?

Interestingly, they will trade but very little changes. Equilibrium markup and output are the same as before. Number of firms is $n = L(1 - \theta)/\alpha$, so more firms in bigger region.

Individuals in home country spend $n/(n + n^*)$ on goods produced at home and wages are equal in both countries. Anything different?

Notice that compared to autarky consumers are better off through gains from variety.
Effect of Trade with Transportation Costs

Iceberg trade costs: a constant percentage of the product $1 - g$ melts on way to other region so that only $g < 1$ percent arrives

How will this affect equilibrium?

Again, firm mark-up is same, $p = w \times \beta / \theta$ and equilibrium number of firms follows same equation $n = L(1 - \theta) / \alpha$

But, wages will be higher in region with larger population!

Very mysterious result. Proved in paper using balance of payments: payments on imports must equal revenue from exports.

However, there is no closed form solution, makes intuition difficult.

Not trivial: consumers in larger region are better off both through greater variety at cheaper cost and because higher wage buys more imported goods
Summarizing results

We have three results to think about:

- In autarky, larger market has more firms but wages and prices can be equal in both markets (not separately identified).

- When regions can trade *without* transportation costs, wages and prices can be the same in both markets. This we can think of as just merging the regions—there is no advantage to any region since goods are freely shipped.

- With transportation costs, bigger region is more attractive to firms because they can sell to home market without transportation costs.

Note that wages and prices are not uniquely defined; saying wages are higher in the larger country same as saying prices are higher: \( \omega = \frac{w}{w^*} = \frac{p}{p^*} \).
**Intuition for balance of payments result**

Easier to think about *prices* adjusting to make balance of payments zero

If the prices were the same then the larger country would have greater export revenue than import costs. This is because they have more domestic products and domestic consumers consume larger quantities of goods *without shipping costs*

Notice that the shipping costs are the key: with no shipping costs number of products in each country is irrelevant. As shipping costs increase, the (equilibrium) relative price in the larger country will increase.

As the price of the larger country increases, domestic goods increase in price relative to foreign imports, increasing import consumption and decreasing domestic consumption

In equilibrium the price rises high enough so that balance of payments is zero
Home Market Effect with Two Industries

Next Krugman allows for two different types of industries where $\alpha$ consumers purchase $\alpha$ industry goods and $\beta$ purchase $\beta$ goods.

This allows for different distributions of tastes, so home market effect now extends to types of industries.

Using similar logic as before, Krugman shows that when tastes are dissimilar enough, countries will specialize in industries where they have a larger home market.

If tastes are below this dissimilarity threshold, both regions produce both goods but region with larger home taste is still net exporter of good.
Paul Krugman’s most famous paper

Contributions of JPE 1991 according to Behrens and Robert-Nicoud:

1. understand how market size influences location choices and location choices influence market size (circular)
2. shows mechanism leading to persistence and path dependence
3. connects location theory to trade theory
4. provided a framework (NEG) applicable to many contexts (ex: taxes) and flexible to incorporate many extensions

Balwin et. al. also note that this paper has “catastrophic agglomeration,” meaning a small change in an underlying parameter suddenly moves the system to a very different state. This is unlike most economics models, but commonly found in models of physical processes (ex: plate tectonics).
Main Questions of Paper

What is the main question of this paper?

Why is manufacturing concentrated and specifically, can we explain this with the interaction of transportation costs and economies of scale?

Sub-questions:

- How sensitive are concentration results to transportation costs?
- What is overall effect of divergence forces (access to bigger market, lower transportation) and convergence forces (competition among firms for fixed expenditure)?
- When will all manufacturing be concentrated in one region? Does the starting point matter?
Basic Setup

- There are two regions and two sectors, farming and manufacturing, which employ farmers and manufacturing workers.
- Farmers are immobile but manufacturing workers can migrate costlessly across regions.
- Trade: the farm good (numeraire) can be shipped to other region costlessly but manufacturing goods have iceberg transport costs: if you send 1 unit of a good from region 1 to region 2 it melts to only $\tau$ units when it arrives in region 2, $\tau < 1$.
- Goal: characterize equilibrium in terms of manufacturing workers in each region; this will provide insight into determinants of concentration.
Consumption

All residents (farmers and workers) have two-level utility function:

\[ U = C_M^{\mu} C_A^{1-\mu} \tag{1} \]

\[ C_M = \left[ \sum_{i=1}^{N} c_i^{\sigma-1} \right]^{\sigma-1} \tag{5} \]

\( N \) is number of potential products, \( \sigma > 1 \); notice consumers have taste for variety

Why two-level? Parameter \( \mu \) determines expenditure on manufacturing, key to concentration
Production

Farmers: one unit of farm labor produces one unit of agricultural good

Supply of farmers is fixed and equal in both regions: \((1 - \mu)/2\)

Workers are mobile but define \(L_1\) and \(L_2\) as workers in each region

\[ L_1 + L_2 = \mu \]  

Production function of manufacturing good \(i\): \(x_i = \frac{L_{Mi} - \alpha}{\beta}\), or

\[ L_{Mi} = \alpha + \beta * x_i \]  

Notice: fixed cost generates IRS and that will vary with wage rate
Optimal price

Symmetry will yield that price is the same for all varieties

Let $p_1$ be price of representative variety in region 1, $w_1$ manufacturing wage. Optimization gives:

$$p_1 = \frac{\sigma}{\sigma - 1} \beta * w_1$$  \hspace{1cm} (5)

Region 2 has similar equation, thus:

$$\frac{p_1}{p_2} = \frac{w_1}{w_2}$$  \hspace{1cm} (6)

Free entry gives zero profit: $\pi = 0$:

$$(p_1 - \beta * w_1)x_1 = \alpha * w_1$$  \hspace{1cm} (7)
Number of varieties

Zero profit condition and price ratio across regions $p_1/p_2 = w_1/w_2$ gives:

$$x_1 = x_2 = \frac{\alpha(\sigma - 1)}{\beta}$$  \hspace{1cm} (8)

Notice that output of any good does not depend on any region specific variables.

With output of each firm we can figure out labor requirement of each firm, which is $\alpha\sigma$.

Then number of firms in a region is total labor divided by per-firm labor, $n_1 = L_1/\alpha\sigma$, and:

$$\frac{L_1}{L_2} = \frac{n_1}{n_2}$$  \hspace{1cm} (6)
Elasticity of substitution measures economies of scale

Turns out that $\sigma$ shows $\text{MPL/APL}$

$\text{MPL} = \frac{1}{\beta}$, $\text{APL} = \frac{\sigma - 1}{\beta \sigma}$

$\frac{\text{MPL}}{\text{APL}} = \frac{\sigma}{\sigma - 1}$

Smaller $\sigma$ gives greater economies of scale ($\text{MPL/APL}$ larger)

Why? As $\sigma$ decreases consumer wants to consume more and more varieties ($\sigma = 1$ is Cobb-Douglas)

Low $\sigma$ leads to more firms, lower output per firm, less labor per firm, lower APL

Note: this is a bit confusing since firms are more productive (lower AC) with higher $\sigma$; however, lower $\sigma$ leads to greater agglomeration economies (same as models with CES production and intermediate input sharing)
Short-run Equilibrium

In short-run workers in each region can’t migrate, want to look at wages

Define $c_{11}$ as *total* consumption in region 1 of a representative region 1 good and $c_{12}$ is region 1 consumption of representative region 2 good

Region 2 must ship $1/\tau$ units so that 1 unit arrives in region 1, thus region 1 consumers pay $p_2 \times (1/\tau)$ for one unit

In region 1, ratio of demand for good 1 to good 2 is:

$$\frac{c_{11}}{c_{12}} = \left(\frac{p_1 \tau}{p_2}\right)^{-\sigma} = \left(\frac{w_1 \tau}{w_2}\right)^{-\sigma}$$  \hspace{1cm} (10)

This equation comes from demand function and equation 6
Expenditure Ratios

Define $z_{11}$ as ratio of total region 1 expenditure on region 1 goods to region 1 expenditure on region 2 goods

$$z_{11} = \left( \frac{n_1}{n_2} \right) \left( \frac{p_1 \tau}{p_2} \right) \left( \frac{c_{11}}{c_{12}} \right) = \left( \frac{L_1}{L_2} \right) \left( \frac{w_1 \tau}{2_2} \right)^{-(\sigma-1)} \quad (11)$$

1. A one percent increase in relative prices reduces quantities sold by $\sigma$ (eq 10) but reduces value by only $\sigma - 1$; basically $p$ is higher in $p \ast q$

2. As a region’s *number* of goods increases overall expenditure share $z_{11}$ also increases
Closing Model

To close model write equations for both regions where total income has to be equal to total expenditure

Define $Y_1$ and $Y_2$ as total income in a region

$$Y_1 = \frac{1 - \mu}{2} + w_1 L_1$$  \hspace{1cm} (15)

Then we have:

$$w_1 \times L_1 = \mu \left[ \left( \frac{Z_{11}}{1 + Z_{11}} \right) Y_1 + \left( \frac{Z_{12}}{1 + Z_{12}} \right) Y_2 \right]$$  \hspace{1cm} (13)
Set of equations *determine* wages but no closed form solution (!); increases difficulty of model

Because many results echo 1980 AER paper he doesn’t discuss in detail

Main focus in short-run is ratio of wages in regions $w_1/w_2$:

1. Increase in manufacturing employment raises utility (lowers variety-adjusted price index) because allows for greater number of firms

2. Increase in employment also can raise wages by more than proportionally increasing output: home market effect

3. However, also a competitive effect working in opposite direction—workers have to share limited amount of peasant expenditure
Long-run Equilibrium: Choice of Units

In long-run we allow workers to migrate

Krugman carefully chooses units to simplify this problem (but without mentioning this, ridiculously confusing!)

\[ \beta = \frac{\sigma - 1}{\sigma} \quad \text{and} \quad \alpha = \frac{\mu}{\sigma} \]

Since \( p_1 = \frac{\sigma}{\sigma - 1} \beta \cdot w_1 \) and \( n_1 = \frac{L_1}{\alpha \cdot \sigma} \)

This implies that \( p_1 = w_1 \) and \( n_1 = L_1 / \mu \)
Long-run Equilibrium: Price Indices

With CES a price index is cost of purchasing one unit of *composite* good at optimal consumption of each variety

\[ P = \left[ \sum_{i=1}^{N} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \]  \hspace{1cm} \text{(N1)}

Price of consumption in region 1 includes local goods and imports

Let \( f \) be fraction of total workers in region 1, \( f = L_1 / \mu \), then using choice of units:

\[ P_1 = \left[ f w_1^{-(\sigma-1)} + (1 - f) \frac{w_2^{-(\sigma-1)}}{\tau} \right]^{\frac{-1}{\sigma-1}} \]  \hspace{1cm} \text{(17)}
Real Wages

Workers migrate based on real wages: what they can consume given region’s nominal wage.

To calculate real wages we need a cost-of-living index: cost of a given level of utility in a region.

CES has a convenient form because expenditure measures utility: $P^\mu \ast P_A^{1-\mu}$

Then real wages are:

$$\omega_1 = \frac{w_1}{P_1^\mu}$$  (19)
Equilibrium Comparative Statics

Main goal is to explain concentration across regions: when will most workers concentrate in one region (“core”) with small region (“periphery”) versus more equal sized regions?

Key is to know how real wage ratio $\omega_1/\omega_2$ varies with share of labor force $f$; if real wage increases with migration then feedback effects will lead to core-periphery equilibrium.

Set of non-linear equations makes analytical solution difficult; instead uses numerical exercises to illustrate main idea.

Three parameters:

1. Share of consumer budget spent on composite good $\mu$.
2. Transportation cost $\tau$—transportation cost cost increases when $\tau$ decreases.
3. Elasticity of substitution $\sigma$, which can measure economies of scale: smaller $\sigma$ greater economies of scale.
Effect of concentration on wage ratio varies by transport cost

Figure 1 makes the point. It shows computed values of $\frac{W_1}{W_2}$ as a function of $f$ in two different cases. In both cases we assume $C = 4$ and $t = .3$. In one case, however, $T = .5$ (high transportation costs); in the other, $T = .75$ (low transportation costs). In the high-transport-cost case, the relative real wage declines as $f$ rises. Thus in this case we would expect to see regional convergence, with the geographical distribution of the manufacturing following that of agriculture. In the low-transport-cost case, however, the slope is reversed; thus we would expect to see regional divergence.

It is possible to proceed entirely numerically from this point. If we take a somewhat different approach, however, it is possible to characterize the properties of the model analytically.
When will industry concentrate?

Depends on interaction of key parameters: $\tau$, $\mu$, $\sigma$

Dispersion: high transportation cost, low manufacturing consumption share, weak economies of scale (large $\sigma$)

Concentration: low transportation cost, high manufacturing consumption share, large economies of scale (small $\sigma$)

Next figure draws boundaries for convergence (concentration): above the line leads to concentration
Boundaries for concentration equilibrium

Increasing $\tau$ is decreasing transportation cost; increasing $\mu$ is increasing manufacturing consumption share.

**Fig. 3**

- $\sigma = 10$
- $\sigma = 4$
When $\nu < 1$ manufacturing will concentrate
Main Forces

Pecuniary Externalities: no direct spillovers in terms of productivity but larger markets have important effects (ex: raise demand, allow more varieties, increase utility)

IRS and taste for variety: leads to one firm per type

Home Market Effect: firms want to locate in larger markets

Can sell to domestic consumers without transportation costs (demand higher, price index lower)

This effect exists even without allowing for mobile workers; in Faber paper connecting two asymmetric regions leads to greater concentration in bigger region (firms ship goods to smaller region)

Reinforcement: given parameters, mobile workers can reinforce home market effect by moving to bigger region, increasing market size with own consumption