Homework 1: Monocentric City Model
Date Assigned: 3/16/2022
Date Due: 3/30/2022
Graduate Urban Economics, SUFE

In this homework I ask you to derive some of the basic results of the monocentric city model and answer a few questions. The setup is identical to that discussed in class (simple, linear city model):

1. Consumers have preferences over housing, q, and the numeraire good z: $U(q, z) = q^{\beta} z^{1-\beta}$
2. All consumers work in the CBD, earn wage w, and commute with per unit distance cost τ; a consumer at x has $\tau * x$ commuting cost
3. Let $P(x)$ and $R(x)$ stand for the housing price and land rent at distance x from the CBD
4. Land is owned by absentee landlords and rented to perfectly competitive developers. Developers have housing production function $H = K^{1-\alpha} * L^{\alpha}$; all land is developable. The cost of capital K is i and land L is rented at rate $R(x)$.
5. Simplest spatial structure: the city is represented by a line (one-dimensional, not two) with the CBD at the left-most endpoint

Problem 1: Derive the Alonso-Muth condition showing the gradient (slope) of the price function, $\frac{\partial P(x)}{\partial x}$ and explain the sign (is the gradient positive, negative, zero, or unknown).

Problem 2: How does consumption of the numeraire change with distance from the CBD? Derive the gradient of numeraire consumption and give a brief interpretation (1-2 sentences is fine).

Problem 3: Let $S(x) = K(x)/L$ represent the capital-land ratio, derive the capital-land gradient $\frac{\partial S(x)}{\partial x}$ and give its sign.

Problem 4: Derive the density gradient and briefly interpret: how does density change as we move away from the city and why? Note: it may help you to first derive the land rent gradient, $\frac{\partial R(x)}{\partial x}$.

Problem 5, Closed City Model: To save you some tedious algebra I will give you specific parameters for this question: $\beta = 0.5$, $w = 100$, $\tau = 20$, $i = 0.1$, $\alpha = 0.75$, $R_A = 100$, $N = 1000$. Calculate the equilibrium utility $\bar{u}$, the distance to the city fringe $\bar{x}$, the land rent at the CBD $R(0)$, the housing price at the CBD $P(0)$, and the density at the CBD $D(0)$ and at the fringe $D(\bar{x})$. Note: these numbers will not work out cleanly and you will need a calculator to solve.

Problem 6, Open City Model: In this question I want you to explain the effect of a transportation cost increase on the density gradient for A) the closed city model and B) the open city model. You do not have to do any calculation, just explain what should happen and why the effect of a transportation cost differs for the two situations (closed vs open).