The Microfoundations of Urban Agglomeration Economies: Dicussion of Duranton and Puga (DP), 2004

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Why Do We Have Cities?

Why are economic activities (and people) clustered rather than completely spatially dispersed?

Why don't we just have a system where everyone is an island economy, consuming their own production?

Starrett Impossibility Theorem (JET 1978, restated in Ottaviano and Thisse 2004):

Consider an economy with a finite number of locations and a finite number of consumers and firms. If space is homogeneous, transport is costly and preferences are locally nonsatiated, then there is no competitive equilibrium involving transportation.

DP: "Without some form of increasing returns we cannot reconcile cities with trade."

Sufficient Conditions for Spatial Clustering

In order to have economic activity cluster it must be either (Ottaviano and Thisse 2004):

1. Space is heterogeneous
2. There are externalities (production or consumption)
3. Markets are imperfect

Today we focus on mechanisms generating increasing returns in cities

Question: if population generates increasing returns why do we have multiple cities?

From Agglomeration to Urban Structure

Fundamental trade-off in cities: increasing returns vs congestion

In monocentric city model we *assumed* people live in cities and did comparative statics with transportation cost

DP embed microfoundations of agglomeration into monocentric model to explain why live in cities

Can then look at how different mechanisms yield predictions about city distribution and production specialization

Three Basic Urban Agglomeration Mechanisms

Urban agglomeration economies according to Marshall:

1) knowledge spill-overs, 2) linkages between input supplies and final good producersand 3) labor market interactions

Many current papers actually use this terminology to describe wide range or agglomeration models

DP instead focus on three basic theoretical mechanisms of most models

1. Sharing
2. Matching
3. Learning

Agglomeration Mechanisms: Sharing

Sharing: Gains from Intermediate Variety

Marshall (1890): even when all firms have CRS production functions, externalities can lead to aggregate increasing returns

DP model:

- Many intermediate producers, each with a fixed cost
- This fixed cost, or indivisibility, leads to IRS
- Final good producer with CRS and CES production function
- Key mechanism: final good production, fixing total amount of input, has IRS wrt number of firms
- Larger cities can support more intermediate firms, increasing productivity of final good

Access to wider variety of inputs (“sharing inputs”) leads to IRS

Final Good Production

Final production in sector j (one producer or aggregate):

$$Y^j = \left[\sum_{h=1}^{n^j} (x_h)^{\frac{1}{1+\epsilon^j}} \right]^{1+\epsilon^j} \tag{1}$$

All inputs are sector-specific (indexed by jh)

Why does this production function have both CRS and IRS properties?

Brief Digression on CES Production

CES production function is quite important in many different fields, including labor (ex: Card and Lemieux, QJE 2001, model production with skilled and unskilled workers), Trade, IO, Urban, and many subfields of Macro

Difficulty with CES is there's lots of algebra, best to start with simple two input production function

$$Y = \left[\alpha_1 * x_1^{\rho} + \alpha_2 * x_2^{\rho}\right]^{\frac{1}{\rho}}$$

The parameter ρ is a measure of substitutability: can you find values of ρ that make this into particular well-known types of production functions?

Elasticity of Substitution for Production

The elasticity of substitution σ for a two input production function is defined as

$$\sigma = \frac{d(x_2/x_1)}{(x_2/x_1)}\frac{TRS}{d\ TRS} = \frac{d\ ln(x2/x1)}{d\ ln|TRS|}$$

For $Y = \left[\alpha_1 * x_1^{\rho} + \alpha_2 * x_2^{\rho}\right]^{\frac{1}{\rho}}$, the TRS is:

$$TRS = -\frac{\alpha_1}{\alpha_2}\left(\frac{x_1}{x_2}\right)^{\rho-1}$$

Take absolute value of TRS, rearrange, and take logs:

$$ln(x_2/x_1) = \frac{1}{1-\rho}ln(|TRS|) + \frac{1}{1-\rho}ln(\alpha_2/\alpha_1)$$

Thus elasticity of substitution is $\sigma = \frac{1}{1-\rho}$

Isoquants for Two Input CES Production

The best way to understand the role of ρ (or σ) is to examine the isoquants because the elasticity of substitution can be interpreted as a measure of isoquant curvature

The function $Y = \left[\alpha_1 * x_1^\rho + \alpha_2 * x_2^\rho\right]^{\frac{1}{\rho}}$ can always be written as (see Varian grad book):

$$Y = A(\rho)\left[\theta * x_1^\rho + (1-\theta) * x_2^\rho\right]^{\frac{1}{\rho}}$$

Graph an isoquant in a simple program (set $A = 1$) and vary ρ (I like mathstud.io—entirely web-based, no need to install anything)

Also, graph an isoquant for the two-input Duranton and Puga specification:

$$Y = \left[x_1^{\frac{1}{1+\epsilon}} + x_2^{\frac{1}{1+\epsilon}}\right]^{1+\epsilon}$$

▸ mathstud.io code

Elasticity of Substitution and Functional Form

The elasticity of substitution for ρ specification: $\sigma = \frac{1}{1-\rho}$

When $\rho = -\infty$, $\sigma = 0$, and we have Leontief production (perfect complements)

When $\rho = 0$, $\sigma = 1$, and we have Cobb-Douglas (at the limit)

When $\rho = 1$, $\sigma = \infty$, and we have linear production (perfect substitutes)

Usually don't allow $\rho > 1$, which leads to concave isoquants and a *negative* elasticity of substitution

In DP framework, $\epsilon > 0$ and elasticity of substitution is $\frac{1+\epsilon}{\epsilon}$

When $\epsilon = 0$ we have perfect substitutes (but assume $\epsilon > 0$) and $\epsilon = \infty$ yields Cobb-Douglas

Higher values of ϵ imply *less* substitutability and thus a greater benefit from increasing the number of inputs

Back to DP 2004: Intermediate Good Production

In a given sector (dropping superscript *j*) output of an intermediate good x_h is:

$$x_h = \beta^j * l_h - \alpha^j \tag{2}$$

Production has IRS in only input: labor (l_h)

No economies of scope, infinite number of potential varieties *h*

Given this setup, how many firms will produce x_h?

Input Demand

We minimize the cost of final good production to derive input demand

Let q_h^j be price of input h in sector j, then cost is:

$$\sum_{h=1}^{n^j} q_h^j * x_h^j$$

Minimizing s.t. producing Y^j yields demand in sector j for input h:

$$x_h^j = \frac{(q_h^j)^{-\frac{1+\epsilon^j}{\epsilon^j}}}{\left[\sum_{h=1}^{n^j}(q_h^j)^{-\frac{1}{\epsilon}}\right]^{1+\epsilon^j}} * Y^j \quad (3)$$

Equilibrium Price

Given a large number of firms we assume no strategic price-setting

Then each firm faces constant own-price elasticity of demand: $-(1+\epsilon^j)/\epsilon^j$

We can see this from eq 3) or ratio of FOC for inputs h and 1 ($h \neq 1$):

$x_h = (\frac{q_h}{q_1})^{-\frac{1+\epsilon^j}{\epsilon^j}} * x_1$

Then intermediate firms set price to maximize profit with a constant mark-up rule

If w^j is labor wage (only input) and given symmetry and identical firms, price is:

$$q^j = \frac{1+\epsilon^j}{\beta^j} * w^j \quad (4)$$

Equilibrium Outpt and Number of Intermediates

Free entry/exit gives zero profit: $q^j x^j - w^j l^j = 0$

Labor as function of output (mistake in footnote 8): $l^j = (x^j + \alpha^j)/\beta^j$

Then zero-profit output with optimal price q^j is:

$$x^j = \frac{\alpha^j}{\epsilon^j} \tag{5}$$

Labor requirement is thus $l^j = \alpha^j(1+\epsilon^j)/(\beta^j\epsilon^j)$, given L^j exogenous total labor:

$$n^j = \frac{L^j}{l^j} = \frac{\beta^j\epsilon^j}{\alpha^j(1+\epsilon^j)} * L^j \tag{6}$$

Localization Economies

Plugging in number of firms and choosing units gives aggregate production:

$$Y^j = \left[n^j (x^j)^{\frac{1}{1+\epsilon^j}} \right]^{1+\epsilon^j} = (L^j)^{1+\epsilon^j} \tag{7}$$

Larger cities have more laborers L^j, leads to more intermediate firms L^j/l^j, leads to more productive final output

This is a very commonly used mechanism; can also be used on the *demand side*

Generally refer to agglomeration economies resulting from proximity to other firms *in same sector* as “localization economies”

Putting into Monocentric City

Same basic set-up as in class but:

- Lot-size (housing consumption) is fixed
- Transport cost in terms of time (more commuting, less time available for work
- Housing rent income is divided among residents (no absentee landlords)
- Workers are free to move across cities *and* sectors
- Wages are endogenous

Monocentric City Set-up

$$U_i^j = w_i^j(1 - 4\tau s) - R_i(s) \tag{1}$$

Worker in city i sector j chooses location s to maximize utility (no choice of housing consumption since lot size is fixed)

Further, fixed lot size implies edges of city are distance $N_i/2$ from CBD

In equilibrium rent adjusts so that there is a resident at all locations $[-N_i/2, N_i/2]$

To find total labor supply in the city (over all sectors) we just integrate total working time (1 unit minus commuting) over the length of the city

Solving Model

$$\sum_{j=1}^{m} L_i^j = N_i(1 - \tau * N_i) = 2 * \int_{s=0}^{N_i/2} (1 - 4\tau s) ds \quad (9)$$

Zero profit $w_i^j * L_i^j = P^j Y_i^j$ and optimal labor $Y_i^j = (L_i^j)^{1+\epsilon^j}$ imply:

$$w_i^j = P^j (L_i^j)^{\epsilon^j} \quad (10)$$

Cities will specialize in just *one* sector, why?

Specialization

Additional workers in a sector j increase the number of intermediate suppliers in sector j ($n^j = L^j / l^j$), which increases productivity ("localization economies")

However, additional workers also raise commuting costs, therefore additional workers in sector k only have a negative effect on j

Consider a small increase in j workers in a city with both j and k: this raises productivity in j, which raises wages, which should attract more j workers. Meanwhile, sector k workers have no increase in productivity but do pay higher commuting costs.

Only stable point is where each city specializes in just one sector

Migration and Optimal City Size

Housing rent adjusts to perfectly offset commuting time; everyone has equal non-housing consumption

Therefore we can measure city utility as a resident's consumption expenditure:

$$c_i^j = P^j (N_i^j)^{\epsilon^j} * (1 - \tau * N_i^j)^{1+\epsilon^j} \quad (11)$$

Consumption-maximizing city size:

$$N^{j*} = \frac{\epsilon^j}{(1 + 2\epsilon^j)\tau} \quad (12)$$

Migration and Stability

$$c_i^j = P^j (N_i^j)^{\epsilon^j} * (1 - \tau * N_i^j)^{1+\epsilon^j} \tag{11}$$

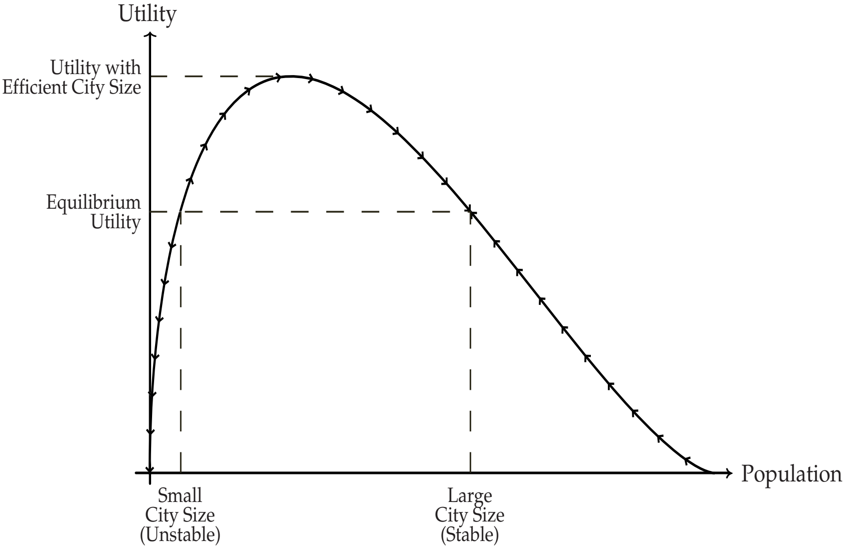
$$\frac{\partial c_i^j}{\partial N_i^j} = P * \left[\epsilon * N^{\epsilon - 1}(1 - \tau * N) + N^{\epsilon}(1 + \epsilon)(1 - \tau * N)^{\epsilon} * -\tau \right] \tag{NS1}$$

Population has two competing effects on utility (consumption): agglomeration and congestion, which lead to hump-shaped curve

Spatial equilibrium (free mobility) implies that all cities must offer workers same consumption

Hump-shape means two equilibrium points at national consumption level; only inefficient point is stable

City Size Diagram



Results

1. Cities will specialize in just one sector
2. All cities of same specialization j will have same population
3. City population will be larger than efficient population

Why inefficient?

Coordination failure prevents development of new cities–no one will move to a new city alone

Additional Discussion of Sharing Mechanism

- Can use a similar model to show gains from *individual* specialization
- However, we fix *n* but assume “learning by doing,” gives IRS in specialization
- Can also have models of risk-sharing: firms working in thick labor market can better adjust hiring to demand shocks

Agglomeration Mechanisms: Matching

Matching in Cities

Larger cities can increase quality of matches between two economic agents

May also increase probability of matching at all (higher number of matches)

Usually consider firms and workers but also marriage market, consumer retail, entrepreneurs, etc....

DP use Salop circular city model to show larger cities have:

1) higher quality of matches 2) more productive firms (IRS)

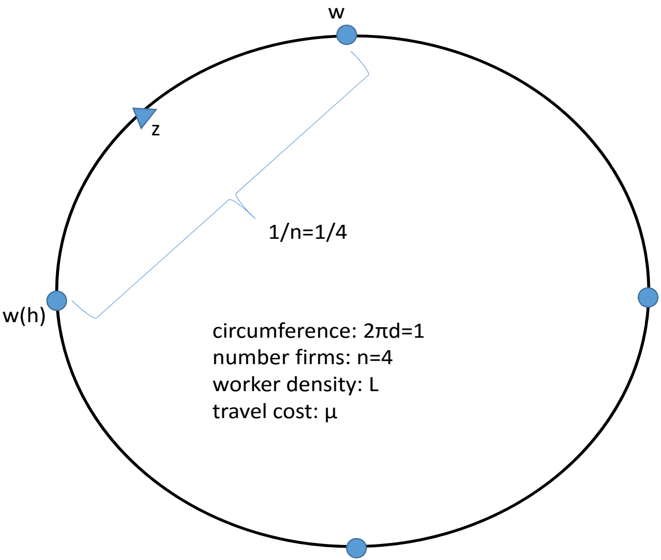
Note: Salop model is simple and applicable in many contexts, quite useful!

Basic Idea of Model

Won't discuss all details (possibly cover more thoroughly in Schiff 2015)

- Workers distributed uniformly around unit circle
- Firms distributed symmetrically around circle
- Distance between firm and worker is measure of skill mismatch; worker pays cost of mismatch
- Firms have IRS production, limited monopsony power
- Free entry and zero profit gives endogenous number of firms and wage

Basic Salop Framework



Firm Profit Maximization

Firm production (IRS): $y(h) = \beta * l(h) - \alpha$

Indifferent worker:

$$w(h) - \mu * z = w - \mu * (\frac{1}{n} - z) \quad (25)$$

Labor is function of own and competing wage:

$$l(h) = 2 * L * z = \frac{L}{n} + [w(h) - w]\frac{L}{\mu} \quad (26)$$

Profit (numeraire): $\pi = (\beta - w(h) * l(h)) - \alpha$

FOC: $\frac{\partial \pi}{\partial w(h)} = 0$, $w(h) = \frac{\beta + w - \frac{\mu}{n}}{2}$

Symmetric Nash Equilibria

Assume symmetry, $w(h) = w$, $w = \beta - \frac{\mu}{n}$

Free entry and zero profit condition gives n:

$\pi = 0$, $l(h) = \frac{L}{n}$, $n = \sqrt{\frac{\mu * L}{\alpha}}$

Total output now IRS in L:

$$Y = n * (\beta * L - \alpha) = \left(\beta - \sqrt{\frac{\alpha * \mu}{L}}\right) * L$$

Expected net wage: $E(w) = \beta - \frac{\mu}{n} - \mu * \frac{1}{4 * n} = \beta - \frac{5}{4}\sqrt{\frac{\alpha * \mu}{L}}$

Comparative Statics

Main question: what happens as population increases?

1. Larger population increases number of firms, but *less than proportionally*
2. Therefore bigger cities have larger, more productive firms (IRS)
3. More firms leads to greater competition for workers $\Rightarrow$ higher wages
4. Av. distance between firm and worker declines, less overall mismatch

DP emphasize that urban agglomeration economies arise not only from IRS, but also due to less mismatch (both factors increase wages)

Additional Discussion of Matching

DP embed Salop model into same monocentric system of cities framework

Most results match sharing except that now coordination to create cities still doesn't achieve efficiency

Reason: efficient number of cities is conditional on number of firms in a city, but Salop set-up leads to excessive entry

However, unclear about generality of this result

Many other matching models; general conclusion is that larger cities increase matching, resulting in higher productivity

Agglomeration Mechanisms: Learning

Learning

Learning in cities is very intuitive; “if you can make it here you can make it anywhere”

However, DP emphasize that in Urban Ec. far less work on how learning drives agglomeration

Types of learning:
1) knowledge generation 2) knowledge diffusion 3) knowledge accumulation

Lots of empirical work in this area right now

Knowledge Generation

Unlike sharing and matching models, knowledge generation often comes from urban diversity

"Urbanization economies": loosely, increases in productivity from proximity to cross-sector factors

Urban diversity (many types of firms and sectors) sometimes source of urbanization economies

Knowledge generation models often have link between diversity and innovation

Modern empirical work in this area tries to show effect of city size, and diversity, on patents

Knowledge Diffusion

DP model uses assumption that probability of learning skill is increasing in a city's population of skilled individuals

This is independent of how many try to acquire skill; perhaps economies of scale in teaching

Use OLG model to show that young live in cities as risky attempt to acquire skills

Skilled workers have incentive to stay in city when old (teach young)

Recent empirical work tries to test “skill acquisition” idea with panel data looking at same worker in different locations (see De la Roca and Puga, ReStud 2016)

Knowledge Accumulation

These models tend to follow growth/macro frameworks

Knowledge accumulates over time in a city, increases productivity

Source of knowledge externality often unspecified, acts like technology in production function

Empirical work: far less, measurement of city-level accumulation seems difficult

Concluding Thoughts

Spatial distribution of population and overwhelming evidence on productivity advantages of cities seems to suggest some kind of externality

These externalities are often vaguely referred to as agglomeration but mechanisms can be quite different

DP framework is useful classification: 1) sharing 2) matching 3) learning

Models presented here provide good framework for thinking about these issues; however, heterogeneity good by assumption

In some cases, can be used more directly in empirical work (ex: variety-adjusted price indices, size of firms, count of varieties)

However, also lots of equivalent predictions which make distinction between mechanisms empirically difficult

Plotting Isoquants in mathsud.io

Basic Two Input Case:

$$Y = \left[\theta * x_1^{\rho} + (1-\theta) * x_2^{\rho}\right]^{\frac{1}{\rho}}$$

```
Slider(t,0,1,0.1)
Slider(p,-10,2,0.05)
Plot(((100^p-t*x^p)/(1-t))^(1/p))
```

Duranton and Puga parameterization:

$$Y = \left[\theta x_1^{\frac{1}{1+\epsilon}} + (1-\theta) x_2^{\frac{1}{1+\epsilon}}\right]^{1+\epsilon}$$

```
Slider(e,-1,1000,0.1)
Slider(t,0,1,0.1)
Plot(((100^(1/(1+e))-t*x^(1/(1+e)))/(1-t))^(1+e))
```

Back