The Microfoundations of Urban Agglomeration Economies: Discussion of Duranton and Puga (DP), 2004

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Shanghai University of Finance and Economics

Graduate Urban Economics, Lecture 3
March 7, 2016
Student Evaluation

Student evaluation: presentation+proposal research idea (40%), midterm (30%), hwk (20%), participation (10%)
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Written proposal: should be a good start to a paper (intro, lit review, early descriptive empirical work or sketch of theory, strategy for challenges)

Homeworks: couple assignments on theory papers, small presentations, referee report
Homework for Next Class

Small (10 minute) presentation on an empirical paper trying to find evidence for agglomeration
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Three groups: 1) sharing 2) matching 3) learning
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Comments?
Why Do We Have Cities?

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Starrett Impossibility Theorem (JET 1978, restated in Ottaviano and Thisse 2004):

*Consider an economy with a finite number of locations and a finite number of consumers and firms. If space is homogeneous, transport is costly and preferences are locally nonsatiated, then there is no competitive equilibrium involving transportation.*
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DP: “Without some form of increasing returns we cannot reconcile cities with trade.”
Sufficient Conditions for Spatial Clustering

In order to have economic activity cluster it must be either (Ottaviano and Thisse 2004):

1. Space is heterogeneous
2. There are externalities (production or consumption)
3. Markets are imperfect

Today we focus on mechanisms generating increasing returns in cities.
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Question: if population generates increasing returns why do we have multiple cities?
From Agglomeration to Urban Structure

Fundamental trade-off in cities: increasing returns vs congestion
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DP embed microfoundations of agglomeration into monocentric model to explain why live in cities
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Can then look at how different mechanisms yield predictions about city distribution and production specialization
Three Basic Urban Agglomeration Mechanisms

Urban agglomeration economies according to Marshall:
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1) knowledge spill-overs
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Sharing: Gains from Intermediate Variety

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Access to wider variety of inputs (“sharing inputs”) leads to IRS
Final production in sector $j$ (one producer or aggregate):

$$Y^j = \left[ \sum_{h=1}^{n^j} \left( x_h \right)^{\frac{1}{1+e^j}} \right]^{1+e^j}$$

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Final production in sector $j$ (one producer or aggregate):

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Why does this production function have both CRS and IRS properties?
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Intermediate Good Production

In a given sector (dropping superscript $j$) output of an intermediate good $x_h$ is:

$$x_h = \beta^j \ast l_h - \alpha^j$$  \hfill (2)
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No economies of scope, infinite number of potential varieties $h$
Intermediate Good Production

In a given sector (dropping superscript $j$) output of an intermediate good $x_h$ is:

$$x_h = \beta^j * l_h - \alpha^j$$  \hspace{1cm} (2)

Production has IRS in only input: labor ($l_h$)

No economies of scope, infinite number of potential varieties $h$

Given this setup, how many firms will produce $x_h$?
Input Demand

We minimize the cost of final good production to derive input demand.
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Let $q^j_h$ be price of input $h$ in sector $j$, then cost is:

$$\sum_{h=1}^{n^j} q^j_h \times x^j_h$$

Minimizing s.t. producing $Y^j$ yields demand in sector $j$ for input $h$:

$$x^j_h = \frac{(q^j_h)^{-\frac{1+e^j}{e^j}}}{\left[ \sum_{h=1}^{n^j} (q^j_h)^{-\frac{1}{e^j}} \right]^{1+e^j}} \times Y^j$$  \hspace{1cm} (3)
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(3)
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Then intermediate firms set price to maximize profit with a constant mark-up rule.
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If \(w^j\) is labor wage (only input) and given symmetry and identical firms, price is:

\[q^j = \frac{1 + \epsilon^j}{\beta^j} \times w^j\] (4)
Equilibrium Outpt and Number of Intermediates

Free entry/exit gives zero profit: $q^j x^j - w^j l^j = 0$
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Labor as function of output (mistake in footnote 8): $l^j = (x^j + \alpha^j) / \beta^j$
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Labor requirement is thus \( l^j = \alpha^j (1 + \epsilon^j) / (\beta^j \epsilon^j) \), given \( L^j \)
exogenous total labor:
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\[
n^j = \frac{L^j}{l^j} = \frac{\beta^j \epsilon^j}{\alpha^j(1 + \epsilon^j)} \times L^j
\]

(6)
Urban Agglomeration Economies

Plugging in number of firms and choosing units gives aggregate production:
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$$Y^j = \left[ n^j (x^j)^{\frac{1}{1+\epsilon^j}} \right]^{1+\epsilon^j} = (L^j)^{1+\epsilon^j}$$ (7)

Larger cities have more laborers $L^j$, leads to more intermediate firms $L^j/l^j$, leads to more productive final output
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This is a very commonly used mechanism; can also be used on the demand side
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Generally refer to agglomeration economies resulting from proximity to other firms in same sector as “localization economies”
Putting into Monocentric City

Same basic set-up as in class but:

- Lot-size (housing consumption) is fixed
- Transport cost in terms of time (more commuting, less time available for work)
- Housing rent income is divided among residents (no absentee landlords)
- Workers are free to move across cities \textit{and} sectors
- Wages are endogenous
Solving Model

\[ \sum_{j=1}^{m} L_{ij} = N_i(1 - \tau \ast N_i) \quad (9) \]

Zero profit \( w_i^j \ast L_i^j = P^j Y_i^j \) and optimal labor \( Y_i^j = (L_i^j)^{1+\epsilon_j} \) imply:

\[ w_i^j = P^j (L_i^j)^{\epsilon_j} \quad (10) \]

- Cities will specialize in just one sector, why? Costs vs benefits
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- Equal housing consumption, housing rent adjusts to perfectly offset commuting time; everyone has equal non-housing consumption
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- Implies total revenue product=total wages=total consumption
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• Cities will specialize in just one sector, why? Costs vs benefits
• Equal housing consumption, housing rent adjusts to perfectly offset commuting time; everyone has equal non-housing consumption
• Implies total revenue product=total wages=total consumption

Therefore we can measure city utility as a resident’s consumption expenditure
Migration and Optimal City Size

\[ c_i^j = P_i^j(N_i^j)^{\epsilon_j} \times (1 - \tau \times N_i^j)^{1+\epsilon_j} \]  \hspace{1cm} (11)

**Consumption-maximizing city size:**

\[ N_i^{j*} = \frac{\epsilon_j}{(1 + 2\epsilon_j)\tau} \]  \hspace{1cm} (12)

There are two effects, agglomeration and congestion, which lead to hump-shaped curve.
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Spatial equilibrium (free mobility) implies that all cities must offer workers same consumption

Hump-shape means two equilibrium points at national consumption level; only inefficient point is stable
The efficient size of a city is the result of a trade-off between urban agglomeration economies and urban crowding. Efficient city size, $N_j^*$ decreases with commuting costs as measured by $\tau$ and increases with the extent of aggregate increasing returns as measured by $\epsilon_j$.

An immediate corollary of this is that the efficient size is larger for cities specialised in sectors that exhibit greater aggregate increasing returns (as argued by Henderson, 1974). In equilibrium, all cities of the same specialisation are of equal size and this size is not smaller than the efficient size. To see this, notice first that cities of a given specialisation are of at most two different sizes in equilibrium (one above and one below the efficient size). This follows from ($11$) and utility equalisation across cities. However, cities below the efficient size will not survive small perturbations in the distribution of workers — as illustrated by the arrows in figure 1, those that gain population will get closer to the efficient size and attract even more workers while those that lose population will get further away from the efficient size and lose even more workers. The same does not apply to cities above the efficient size — in this case, those that gain population will get further away from the efficient size while those that lose population will get closer. The combination of free mobility with a stability requirement therefore implies the result that cities of the same specialisation are of equal size and too large.

The result that cities are too large is the consequence of a coordination failure with respect to $16$ As $\epsilon_j$ increases, the elasticity of substitution across the varieties of intermediate inputs \((1 + \epsilon_j)/\epsilon_j\) falls, so that there is a greater benefit from having access to a wider range of varieties.
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Results

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2. All cities of same specialization \( j \) will have same population
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Why inefficient?
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Why inefficient?

Coordination failure prevents development of new cities—no one will move to a new city alone
Additional Discussion of Sharing Mechanism

- Can use a similar model to show gains from *individual* specialization
- However, we fix $n$ but assume “learning by doing,” gives IRS in specialization
- Can also have models of risk-sharing: firms working in thick labor market can better adjust hiring to demand shocks
Matching in Cities

Larger cities can increase quality of matches between two economic agents
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1) higher quality of matches 2) more productive firms (IRS)

Note: Salop model is simple and applicable in many contexts, quite useful!
Basic Idea of Model

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Basic Idea of Model

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- Workers distributed uniformly around unit circle
- Firms distributed symmetrically around circle
Basic Idea of Model

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- Firms distributed symmetrically around circle
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- Firms have IRS production, limited monopsony power
- Free entry and zero profit gives endogenous number of firms and wage
Basic Salop Framework

- circumference: $2\pi d = 1$
- number firms: $n = 4$
- worker density: $L$
- travel cost: $\mu$

$1/n = 1/4$

$w$

$w(h)$

$z$
Firm Profit Maximization

Firm production (IRS): \( y(h) = \beta \ast l(h) - \alpha \)
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Indifferent worker:

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    w(h) - \mu \ast z = w - \mu \ast \left( \frac{1}{n} - z \right) \quad (25)
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Profit (numeraire): \( \pi = (\beta - w(h) \ast l(h)) - \alpha \)
Firm Profit Maximization

Firm production (IRS): \( y(h) = \beta \times l(h) - \alpha \)

Indifferent worker:

\[
w(h) - \mu \times z = w - \mu \times \left( \frac{1}{n} - z \right)
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FOC: \( \frac{\partial \pi}{\partial w(h)} = 0 \), \( w(h) = \frac{\beta + w - \frac{\mu}{n}}{2} \)
Symmetric Nash Equilibria

Assume symmetry, $w(h) = w$, $w = \beta - \frac{\mu}{n}$
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Free entry and zero profit condition gives \( n \):

\[
\pi = 0, \quad l(h) = L n, \quad n = \sqrt{\mu L^\alpha} \]

Total output now IRS in \( L \):

\[
Y = n \ast (\beta L - \alpha) = (\beta - \sqrt{\alpha L^\mu}) \ast L \]

Expected net wage:

\[
E(w) = \beta - \frac{\mu}{n} - \frac{\mu}{4} n = \beta - \frac{5}{4} \sqrt{\alpha L^\mu} \]
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Free entry and zero profit condition gives \( n \):

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Total output now IRS in \( L \):

\[
Y = n \times (\beta \times L - \alpha) = \left( \beta - \sqrt{\frac{\alpha \mu L}{L}} \right) \times L
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Expected net wage: \( E(w) = \beta - \frac{\mu}{n} - \mu \times \frac{1}{4 \times n} = \beta - \frac{5}{4} \sqrt{\frac{\alpha \mu L}{L}} \)
Comparative Statics

Main question: what happens as population increases?
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DP emphasize that urban agglomeration economies arise not only from IRS, but also due to less mismatch (both factors increase wages)
Additional Discussion of Matching

DP embed Salop model into same monocentric system of cities framework
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Many other matching models; general conclusion is that larger cities increase matching, resulting in higher productivity
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Types of learning:
1) knowledge generation 2) knowledge diffusion 3) knowledge accumulation

Lots of empirical work in this area right now
Knowledge Generation

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Knowledge generation models often have link between diversity and innovation.

Modern empirical work in this area tries to show effect of city size, and diversity, on patents.
Knowledge Diffusion

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Recent empirical work tries to test “skill acquisition” idea with panel data looking at same worker in different locations (see De la Roca and Puga, ReStud 2016).
Knowledge Accumulation

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Empirical work: far less, measurement of city-level accumulation seems difficult
Concluding Thoughts

Spatial distribution of population and overwhelming evidence on productivity advantages of cities seems to suggest some kind of externality.
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In some cases, can be used more directly in empirical work (ex: variety-adjusted price indices, size of firms, count of varieties).

However, also lots of equivalent predictions which make distinction between mechanisms empirically difficult.