System of Cities Models:
Discussion of Henderson 1974 and DP 2013

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Administration

Main Questions

Models following Henderson (AER 1974) seek to answer a series of related questions:

1. What explains the size of a city?
2. Why do cities have different sizes?
3. Why do cities have different industry specializations?
4. Is there an efficient (optimal) city size?
5. Can we reach the efficient size without coordinated city creation?
6. If instead we have city creation, how do private city developers compare to local governments?

Main Ideas

1. Effect of population size on output reflects tradeoff between agglomeration and congestion
2. Agglomeration occurs only *within* sectors (sector employment size), congestion occurs across sectors (total population)
3. Within sector agglomeration makes cities specialize in one sector
4. Different sectors may have different productivities, leads to different optimal and equilibrium population sizes
5. If agglomeration is concave in population and congestion is convex we get an optimal city size at positive population level
6. Cities can be too big because migrants don't consider externalities of location decision, stable point occurs when negative externality outweighs positive externality
7. Private developers and local governments can both coordinate city creation to reach optimal city size

Duranton and Puga 2013

Handbook of Economic Growth article presents version of Henderson's original paper

Somewhat simpler to read, uses CES framework, nests monocentric city model; qualitative points are the same

Uses similar framework to DP 2004 handbook article but much more depth

Next class we read Henderson and Au (2006) article on migration restrictions and city sizes in China

China paper uses model actually closer to DP 2013 version

DP paper also discusses interesting empirical implications of model

CES Production

Model uses familiar CES production function to generate agglomeration ("sharing")

Final good production in city i in sector j:

$$Y_i^j = \beta^j \left\{ \int_0^{m_i^j} \left[y_i^j(h) \right]^{\frac{1}{1+\sigma^j}} dh \right\}^{1+\sigma^j} \tag{5.34}$$

$$y_i^j(h) = \beta^j l_i^j(h) - \alpha^j \tag{5.35}$$

Profit maximization leads to markup rule for price of input j: $q_i^j = \frac{1+\sigma^j}{\beta^j} * w_i^j$

Free entry makes intermediate input supplier profit zero: $q_i^j * y_i^j - w_i^j * l_i^j = 0$

Number of Firms

As in Krugman papers, we find optimal intermediate output is independent of city factors:

$$y_i^j = \frac{\alpha^j}{\sigma^j} \tag{5.36}$$

From optimal output we can derive number of varieties as function of size of labor force:

$$m_i^j = \frac{N_i^j}{l_i^j} = \frac{\beta^j \sigma^j}{\alpha^j (1 + \sigma^j)} * N_i^j \tag{5.37}$$

Agglomeration

Choose units so that $\beta^j = (1+\sigma^j)(\alpha^j/\sigma^j)^{\sigma^j(1+\sigma^j)}$

Then aggregate production in sector j of city i:

$$Y_i^j = \beta^j \left[\left(y_i^j \right)^{\frac{1}{1+\sigma^j}} m_i^j \right]^{1+\sigma^j} = \beta^j \left(N_i^j \right)^{1+\sigma^j} \tag{5.38}$$

Final good producers are perfectly competitive (free entry, multiple cities can produce same product j) and thus have zero profits: $w_i^j N_i^j = P^j Y_i^j$:

$$w_i^j = P^j \beta^j \left(N_i^j \right)^{\sigma^j} \tag{5.39}$$

Note: 5.38 and 5.39 suggest empirical log specification

Specialization

Localization economies imply every city specializes in one sector j, why?

Agglomeration within sector, congestion across sectors

Why then did the Krugman models have multiple sectors in the same city?

Trade: here we have zero trade costs, no home-market effect where local consumers avoid transportation costs

Both Krugman and system of cities models have congestion forces limiting population of city, differences?

Here congestion occurs through greater commuting; Krugman has local firms selling to a fixed number of farmers

Congestion: simplified version

Commuting costs CC have constant elasticity in distance, cost measured in units of city's locally produced good

$CC(x, \tau, P^j) = P^j \frac{1+\gamma}{\gamma} \tau * x^\gamma$ and $d \log(CC)/d \log(x) = \gamma$

Further, let housing size $h(x)$ and capital to land ratio $f(x)$ be fixed: $h(x) = f(x) = 1$

This means everybody lives in a single floor house (no one above them) and fringe is equal to population: $\bar{x} = N_i$

Let $P(x)$ be housing rent; everyone in city has same utility $\bar{v}$:

$$v\left(P(x), w - CC(x)\right) = \bar{v} \tag{5.2}$$

Land Prices and Land Rent Functions

Totally differentiating 5.2 gives land rent and housing rent gradients:

$\frac{dR(x)}{dx} = \frac{dP(x)}{dx} = \frac{dCC(x)/dx}{h(x)} = -P^j(1+\gamma)\tau * x^{\gamma-1}$

Note: $\frac{dR(x)}{dx} = \frac{dP(x)}{dx}$ because of assumption on $f(x)$

Let cost of capital be zero ($i = 0$) and fringe land rent zero $R(\bar{x}) = 0$

Then $\int \frac{dP(x)}{dx} = \int \frac{dR(x)}{dx} = -P^j \frac{1+\gamma}{\gamma} \tau * x^{\gamma} + C$

We know $R(\bar{x}) = 0 = -P^j \frac{1+\gamma}{\gamma} \tau * \bar{x}^{\gamma} + C$ and $\bar{x} = N_i$, which implies

$$P(x) = R(x) = P^j \frac{1+\gamma}{\gamma} \tau * (N_i^{\gamma} - x^{\gamma}) \qquad (5.40)$$

Total Land Rent

With fixed housing consumption, equal utility implies everyone consumes same amount of numeraire

Therefore expenditure on housing and commuting for a person at x must be equal to expenditure on housing for person at CBD, with no commuting:

$P^j \frac{1+\gamma}{\gamma} \tau * x^\gamma + P(x) = P(0) = P^j \frac{1+\gamma}{\gamma} \tau * N_i^\gamma$

We use above expression when discussing resident utility

Finally, integrating 5.40 gives total rents:

$$R_i = \int_0^{\bar{x}=N_i} R(x)dx = P^j \tau * N_i^{1+\gamma} \qquad (5.41)$$

Who Creates Cities?

So far we have a model of productivity and specialization but no way to determine number of cities

Consider three alternative mechanisms: 1) self-organization 2) city-developers 3) local government

We study equilibrium outcomes under these different mechanisms

Important detail: who earns land rent?

Self-organizing Cities

In self-organizing cities there is no coordinating force to create a new city; individual worker is to small to create city on their own

Assume land rents are equally divided among residents: R_i/N_i

Residents have utility only over housing and numeraire, consume equal housing, thus wages net of housing expenditure is sufficient statistic for utility: $c_i = w(N_i) + \frac{R_i}{N_i} - P(0)$

Using 5.39-5.41 we get:

$$c_i = P^j \left(\beta^j * N_i^{\sigma^j} - \frac{\tau}{\gamma} N_i^{\gamma} \right) \tag{5.42}$$

Equilibrium in Self-Organizing Cities

$$c_i = P^j \left(\beta^j * N_i^{\sigma^j} - \frac{\tau}{\gamma} N_i^{\gamma} \right) \tag{5.42}$$

If $\sigma^j < \tau$ utility (consumption) is concave in population

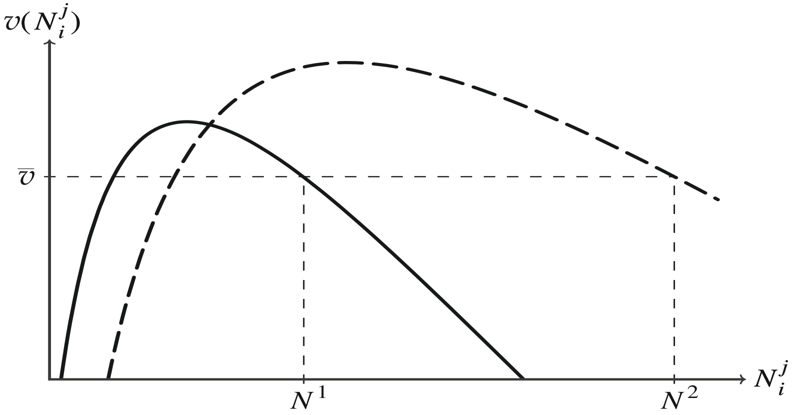
This yields the inverse U graph of utility against population

There will be an optimal city size but the stable size has too many people

$N^* = \left(\beta^j \frac{\sigma^j}{\tau} \right)^{\frac{1}{\gamma - \sigma^j}}$

In equilibrium all cities must offer same utility but cities with different specialities can have different populations

Self-organized City Has Too Many People



Panel (a)
Self-organization

Land Developer

Land developers create new cities and collect all land rents, $R_i = P^j * \tau * N_i^{\gamma+1}$

To induce migrants to come to a new city they can offer a payment of T_i

Land developer profit equation is thus:

$$\max_{T_i, N_i} \Pi_i = P^j \tau N_i^{1+\gamma} - T_i N_i \tag{5.43}$$

However, migrants will not come to a new city unless they receive (at least) the same utility $\bar{c}$—wages minus housing plus commuting costs $(w(N_i) - P(0)$—as other cities

$$P^j \beta^j N_i^{\sigma^j} + T_i - P^j \frac{1+\gamma}{\gamma} \tau N_i^{\gamma} = \bar{c} \tag{5.44}$$

Land Developer's Problem

Minimum utility $\bar{c}$ gives required transfer T_i:

$T_i = \bar{c} + P^j \left[\frac{1+\gamma}{\gamma} \tau N_i^\gamma - \beta^j N_i^\sigma \right]$

Then (unconstrained) profit maximization problem is:

$$\max_{N_i} \Pi_i = P^j \beta^j N_i^{1+\sigma^j} - P^j \frac{\tau}{\gamma} N_i^{1+\gamma} - \bar{c} N_i \qquad (5.45)$$

DP point out that this equation shows land developers acting like owners of a "factory town": revenue is $P^j Y_i^j = P^j \beta^j N_i^{1+\sigma^j}$ and labor costs are $P^j \frac{\tau}{\gamma} N_i^{1+\gamma} + \bar{c} N_i$

Solving for the FOC gives (in DP $\bar{v} = \bar{c}$):

$$\bar{v} = \bar{c} = P^j \left((1+\sigma^j) \beta^j N_i^{\sigma^j} - \frac{1+\gamma}{\gamma} \tau N_i^\gamma \right) \qquad (5.46)$$

Land Developer Internalizes Externality

Put FOC into profit to get maximized profits:

$$\Pi_i = P^j \tau N_i^{1+\gamma} - \sigma^j P^j \beta^j N_i^{1+\sigma^j} \tag{5.47}$$

Then from 5.43 we know $T_i N_i = \sigma^j P^j \beta^j N_i^{1+\sigma^j}$, or

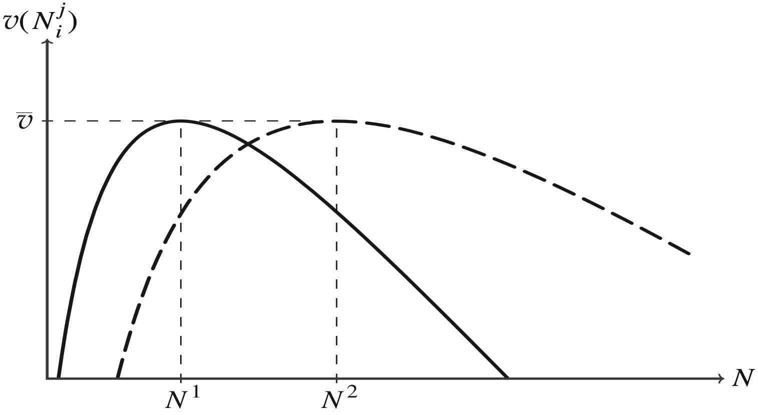
$$T_i = \sigma^j P^j \beta^j N_i^{\sigma^j} \tag{5.48}$$

Finally, with free entry $\Pi_i = 0$ in 5.47, which yields:

$$N_i = \left(\beta^j \frac{\sigma^j}{\tau} \right)^{\frac{1}{\gamma - \sigma^j}} \tag{5.49}$$

This is exactly the utility maximizing population!

Land Developers Can Choose Optimal Size



Panel (b)
Land developers

Free Entry and Equal Utility

Why do all sectors produce at efficient output level?

If not, additional developers could enter and make positive profits by producing at more efficient city size

If some sectors are more productive than others (higher β^j) or have greater agglomeration benefits σ^j won't land developers offer higher utility to residents?

Process: sectors with higher utility will have additional entry, entry increases production of product j, P^j declines, and thus transfers to workers must decline

Implications

1. Improvements in transportation technology (lower γ or τ) will increase city size
2. City size also increases with productivity gains (higher β^j) or stronger agglomeration forces (higher σ^j)
3. In decentralized equilibrium cities will be too large
4. Possibility of using policy to improve welfare
5. Lots of ongoing work: search “Henry George Theorem” and optimal city size